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Buckling of composite cylindrical shells with rigid end disks under hydrostatic pressure

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ABSTRACT

An analytical solution of the buckling problem formulated for a composite cylindrical shell with its ends closed by rigid disks and subjected to hydrostatic pressure is presented in the paper. The problem is solved using Fourier decomposition and the Galerkin method. The boundary conditions are assigned in the form accounting for axial displacements of the end disks caused by an axial contraction of the deformed shell. The hoop displacement and deflection of the shell are approximated by the beam function corresponding to the first mode shape of vibration of a clamped-clamped beam. The axial displacement is approximated by the third derivative of the beam function. Based on this solution, a number of analytical formulas enabling calculations of critical hydrostatic pressure for composite orthotropic cylindrical shells are derived. Using these formulas, the critical loads are calculated for the shells with various elastic and geometric properties. The calculations are verified by comparisons with the results of finite-element analyses. The efficiency of analytical solutions in the search for fibre reinforcement arrangements providing maximum resistance to buckling is demonstrated by several examples.

1. Introduction

Typical designs of underwater vehicles consist of a cylindrical shell with closed ends of various shapes. The ends and cylindrical part are normally joined through a rigid ring. Submerged underwater vehicle is subjected to external hydrostatic pressure. Thus, one of the typical modes of failure of such a structure is the buckling of its cylindrical part. In the buckling analysis, the overall load applied to the cylindrical shell under consideration has two components: (a) a uniform external pressure; and (b) an axial compressive loads uniformly distributed over the edges of the shell at the ends. The latter loads are statically equivalent to the pressure multiplied by the area of the shell cross-section. Studies of buckling of the cylindrical shells subjected to such a combined loading attracted attention of many researchers. Classical solutions of this type of problem formulated for metal shells can be found in the monographs by Flügge [1], Timoshenko and Gere [2], Brush and Almroth [3], Yamaki [4], Volmir [5], Alfutov [6], Jones [7], and Venstel and Krauthammer [8]. Conceptual designs of underwater vehicles made of composite materials are considered in the articles published by Ross [9,10]. Results of buckling analyses of composite cylindrical shells subjected to uniform external pressure can be found in the papers by Carvelli et al. [11], Messager et al. [12], Hur et al. [13], Hernandez-Moreno et al. [14], Moon et al. [15], and Li and Qiao [16]. Finite-element modelling and analysis of laminated axisymmetric shells was reported by Correia et al. [17] and Santos et al. [18]. An analytical solution of the buckling problem for a composite cantilever circular shell subjected to uniform external pressure is presented by the authors in [19]. The Galerkin method is employed to perform buckling analysis of a composite orthotropic cylindrical shell with clamped edges subjected to inertia loading in [20]. An approximate analytical solution of the buckling problem formulated for a composite sandwich cylindrical shell subjected to a uniform external lateral pressure is presented in [21]. The buckling problem of the cylindrical shell loaded by a hydrostatic pressure (i.e. combined external lateral pressure and axial compression) has an analytical solution if the ends of the shell are simply supported. This type of support is characterised by low torsional and bending stiffness of the joining ring and the attached end shell. So the edges of such a shell are compliant with regard to rotation and could be treated as the simply supported. However, in practice, the joining ring has a substantial torsional stiffness which is also reinforced by the stiffness of the shell ends also having considerable bending stiffness. Thus, the rotation of the edges of the cylindrical part of the body is normally restricted under loading.

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This can be modelled, by considering a cylindrical shell with the ends closed by the rigid disks. In the process of deformation, the ends of the shell should be free to move in the axial direction. This type of support must be properly reflected on by the relevant boundary conditions.

The conventional solution algorithm of the buckling problem for cylindrical shells under hydrostatic pressure and boundary conditions different from the simple support is long well known. The description of such a procedure can be found, for example, in the paper by Sobel [22]. The critical pressure is determined by solving a nonlinear equation which is derived using various computational approaches. One of such approaches requires a search of the roots of the characteristic polynomial of the eighths order. The roots found in this process are used to construct the solutions of differential equations modelling the buckling of the shell. Alternative computational procedure requires a calculation of the determinant of the eights order which is formed as a result of realisation of boundary conditions. These computations should be performed for each number of circumferential buckling waves. Clearly, these approaches require a substantial computational effort which makes them inefficient when designing underwater vehicle structures, particularly those made of composite materials. Thus, it is advantageous to have an analytical formula available for such calculations that would facilitate fast and reliable selection of appropriate structural design parameters. Such formulas are derived in this paper. An analytical solution of the buckling problem under consideration is obtained using Fourier decomposition and the Galerkin method. The boundary conditions are assigned in the form accounting for axial displacements of the end disks caused by an axial contraction of the deformed shell. The hoop displacement and deflection of the shell are approximated by the beam functions corresponding to the first mode shape of vibration of a clamped-clamped beam. The axial displacement is approximated by the third derivative of the beam function. A number of analytical formulas providing the ways to calculate critical hydrostatic pressures for composite orthotropic cylindrical shells are derived. Using these formulas, the parametric buckling analyses are performed for the shells having various elastic and geometric properties. The calculations are verified by finite-element analyses. The efficiency of analytical solutions in the design analyses is demonstrated using particular examples in which the structure of composite material is determined to deliver maximum resistance to buckling.

2. Governing equations

Consider a composite orthotropic cylindrical shell of length l. The middle surface of the shell with the radius R is referred to the curvilinear coordinate frame α, β, and γ as shown in Fig. 1. The rigid disks are attached to the ends of the shell at $\alpha = 0,\ l$ (see Fig. 1). The cylindrical surface of the shell and the disks are subjected to the uniform external pressure p. This loading leads to the radial and axial compression of the shell as shown in Fig. 2.

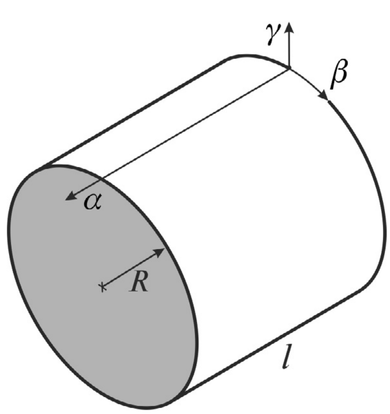

Fig. 1. Cylindrical shell with the ends closed by rigid disks.

Analysis is performed using the following linearized buckling equations of orthotropic cylindrical shells [23]:

$$\begin{aligned}
&\frac{\partial N_\alpha}{\partial \alpha} + \frac{\partial N_{\alpha\beta}}{\partial \beta} = 0\\
&\frac{\partial N_{\alpha\beta}}{\partial \alpha} + \frac{\partial N_\beta}{\partial \beta} + \frac{\partial M_{\alpha\beta}}{R\partial \alpha} + \frac{\partial M_\beta}{R\partial \beta} = 0\\
&\frac{\partial^2 M_\alpha}{\partial \alpha^2} + 2\frac{\partial^2 M_{\alpha\beta}}{\partial \alpha \partial \beta} + \frac{\partial^2 M_\beta}{\partial \beta^2} - \frac{N_\beta}{R}\\
&+N^o_\alpha \frac{\partial^2 w}{\partial \alpha^2} + N^o_{\alpha\beta}\left(2\frac{\partial^2 w}{\partial \alpha \partial \beta} - \frac{\partial v}{R\partial \alpha}\right) + N^o_\beta\left(\frac{\partial^2 w}{\partial \beta^2} - \frac{\partial v}{R\partial \beta}\right) = 0
\end{aligned} \tag{1}$$

constitutive equations

$$\begin{aligned}
&N_\alpha = B_{11}\varepsilon_\alpha + B_{12}\varepsilon_\beta, \quad N_\beta = B_{21}\varepsilon_\alpha + B_{22}\varepsilon_\beta\\
&N_{\alpha\beta} = B_{33}\varepsilon_{\alpha\beta}\\
&M_\alpha = D_{11}\kappa_\alpha + D_{12}\kappa_\beta, \quad M_\beta = D_{21}\kappa_\alpha + D_{22}\kappa_\beta\\
&M_{\alpha\beta} = D_{33}\kappa_{\alpha\beta}
\end{aligned} \tag{2}$$

and strain-displacement relationships

$$\begin{aligned}
&\varepsilon_\alpha = \frac{\partial u}{\partial \alpha}, \quad \varepsilon_\beta = \frac{\partial v}{\partial \beta} + \frac{w}{R}, \quad \varepsilon_{\alpha\beta} = \frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha}\\
&\kappa_\alpha = -\frac{\partial^2 w}{\partial \alpha^2}, \quad \kappa_\beta = -\frac{\partial^2 w}{\partial \beta^2} + \frac{\partial v}{R\partial \beta}, \quad \kappa_{\alpha\beta} = -2\frac{\partial^2 w}{\partial \alpha \partial \beta} + \frac{\partial v}{R\partial \alpha}
\end{aligned} \tag{3}$$

in which N_α, N_β and $N_{\alpha\beta}$ are the longitudinal, hoop, and shear membrane stress resultants; M_α, M_β and $M_{\alpha\beta}$ bending and twisting moments; ε_α, ε_β and $\varepsilon_{\alpha\beta}$ longitudinal, hoop, and in-plane shear strains of the middle surface; κ_α, κ_β and $\kappa_{\alpha\beta}$ bending and twisting deformations of the middle surface; u, v and w in-plane longitudinal and hoop displacements, and deflection of the middle surface; $B_{11}, B_{12}, B_{22}, B_{33}\ (B_{21} = B_{12})$ and $D_{11}, D_{22}, D_{12}, D_{33}\ (D_{12} = D_{21})$ membrane and bending stiffnesses of the shell wall [24]; N^o_α, N^o_β and $N^o_{\alpha\beta}$ membrane pre-buckling forces caused by the hydrostatic pressure.

The system of equations Eqs. (1)–(3) should be supplemented by the boundary conditions at the shell ends, i.e. at $\alpha = 0,\ l$. It is assumed that the rigid disks have a freedom to move in the axial direction remaining parallel to their original position (i.e. orthogonal to the axis of the shell). This can be achieved if the following conditions are satisfied:

$$N_\alpha = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial w}{\partial \alpha} = 0 \tag{4}$$

Also, the pre-buckling state of the shell is assumed to be membrane. The corresponding membrane stress resultants are determined as follows

$$N^o_\alpha = -\frac{N}{2}, \quad N^o_\beta = -N, \quad N^o_{\alpha\beta} = 0 \tag{5}$$

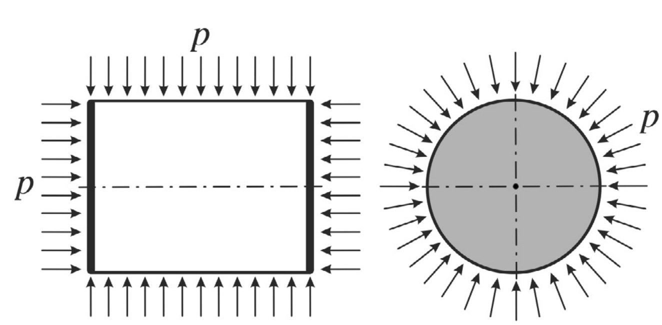

Fig. 2. Cylindrical shell loaded by hydrostatic pressure.