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# On Extension-Shearing Bending-Twisting coupled laminates

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ABSTRACT

This article presents details of the development of a special class of laminate, possessing *Extension-Shearing Bending-Twisting* coupling, necessary for optimised passive-adaptive flexible wing-box structures. The possibility of achieving a measurable drag reduction in cruise flight, without the cost or reliability issues associated with active control mechanisms, is of significant interest for achieving increased fuel burn efficiency, and meeting associated emissions targets. The introduction of passive *Bending-Twisting* coupling at the wing-box level has been previously demonstrated through laminate level tailoring with *Extension-Shearing* coupling only, but the limited design space and the possibility for ply terminations (to produce tapered thickness) effectively rule out this special class of laminate for practical construction. The study is now broadened to consider laminates with *Extension-Shearing Bending-Twisting* coupling, beyond the less well-known un-balanced and symmetric design rule or indeed balanced and symmetric designs with off-axis alignment. Results reveal a vast laminate design space with *Extension-Shearing* coupling that can be maximised without the unfavourable strength characteristics associated with off-axis alignment. Results also reveal that shear buckling strength can be maximised through *Bending-Twisting* coupling when load reversal is not a design constraint.

## 1. Introduction

This article is one in a series, investigating unique forms of thermo-mechanically coupling behaviour for laminated composite materials. These are described collectively in an original article [1], identifying all 24 possible coupling interactions between *Extension*, *Shearing*, *Bending* and *Twisting*.

Here, attention is focussed on the identification of laminated composite materials possessing mechanical *Extension-Shearing Bending-Twisting* coupling, in which the in-plane properties are decoupled from out-of-plane properties. It complements two previous article on isolated mechanical *Extension-Shearing* coupling [2] and isolated mechanical *Bending-Twisting* coupling [3].

The motivation for this study is to explore the potential for maximising *Extension-Shearing* coupling, whilst minimising the detrimental effects of *Bending-Twisting* coupling. It was demonstrated that this can achieved with standard ply orientations used in current industrial design practice, without using off-axis alignment of the principal material axis [4,5]. This approach is known to have a detrimental effect on material strength constraints that must be avoided. Note that whilst *Bending-Twisting* coupling has detrimental effects on compression buckling strength [3], it does have the potential to increase shear buckling strength, as this article will demonstrate.

A complete list of *Extension-Shearing Bending-Twisting* coupled laminate is therefore developed, beyond the less well-known un-balanced and symmetric design rule [6]. The listings contain laminates with up to 21 plies; or 42 plies if the data is interpreted as the symmetric half of the laminate stacking sequence definition.

Laminate stacking sequence configurations are derived in symbolic form together with dimensionless parameters from which the extensional and bending stiffness terms are readily calculated; solutions are therefore independent of the fibre/matrix system and angle-ply orientation.

Unlike many previous studies in the literature, the results are applicable to, and indeed presented as laminate designs containing 0°, 90°, 45° and −45° ply orientations only, which is standard industrial design practice.

Expressions relating the dimensionless parameters to the well-known ply orientation dependent lamination parameters are also given, together with graphical representations of feasible domains for a range of ply number groupings, i.e., a set of laminate designs with a specific number of plies ($n$).

Quasi-Homogeneous laminates are also introduced as an important sub-set of *Extension-Shearing Bending-Twisting* coupled laminates, since such laminates have concomitant orthotropic

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### Nomenclature

| | |
|---|---|
| **A**, $A_{ij}$ | extensional stiffness matrix and its elements |
| **B**, $B_{ij}$ | coupling stiffness matrix and its elements |
| **D**, $D_{ij}$ | bending stiffness matrix and its elements |
| $E_{1,2}$, $G_{12}$ | in-plane Young's moduli and shear modulus |
| $H$ | laminate thickness (=number of plies, $n \times$ ply thickness, $t$) |
| $k_{xy}$ | non-dimensional buckling load factor in shear |
| **M** | vector of moment resultants (=$\{M_x, M_y, M_{xy}\}^T$) |
| **N** | vector of force resultants (=$\{N_x, N_y, N_{xy}\}^T$) |
| $n$ | number of plies in laminate stacking sequence |
| $Q_{ij}$ | reduced stiffness elements |
| $Q'_{ij}$ | transformed reduced stiffness elements |
| $t$ | ply thickness |
| $U_E$, $U_G$ | laminate invariants for equivalent isotropic properties, $E_{Iso}$ and $G_{Iso}$ |
| $U_\Delta$, $U_v$ | laminate invariants for orthotropic properties and Poisson ratio |
| $x,y,z$ | principal axes |
| $z_k$ | layer $k$ interface distance from laminate mid-plane |
| $\boldsymbol{\varepsilon}$ | vector of in-plane strains (= $\{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}^T$) |
| $\boldsymbol{\kappa}$ | vector of curvatures (= $\{\kappa_x, \kappa_y, \kappa_{xy}\}^T$) |
| $\lambda$ | buckling half-wave |

| | |
|---|---|
| $\nu_{ij}$ | Poisson ratio |
| $\theta_k$ | ply orientation for layer $k$ |
| $\xi^A_\Delta$,$\xi^A_v$ | lamination parameters for orthotropic extensional stiffness |
| $\xi^A_{\Delta c}$,$\xi^A_{vc}$ | lamination parameters for coupled extensional stiffness |
| $\xi^D_\Delta$,$\xi^D_v$ | lamination parameters for orthotropic bending stiffness |
| $\xi^D_{\Delta c}$,$\xi^D_{vc}$ | lamination parameters for coupled bending stiffness |
| $\zeta$ | bending stiffness parameter for laminate (= $n^3$) |
| $\zeta_\pm$ | bending stiffness parameter for angle-ply sub-sequence |
| $\zeta_+$,$\zeta_-$ | bending stiffness parameter for positive/negative angle-ply sub-sequence |
| $\zeta_\circ$,$\zeta_\bullet$ | bending stiffness parameter for cross-ply sub-sequences |
| +, − ,± | angle plies, used in stacking sequence definition |
| ○,● | cross-plies, used in stacking sequence definition |

*Matrix sub-scripts*

| | |
|---|---|
| 0 | All elements zero |
| F | All elements Finite |
| I | Isotropic form |
| S | Specially orthotropic or *Simple* form |

properties, i.e., matching orthotropic stiffness in extension and bending. Quasi-Homogeneous designs permit ply percentage and buckling strength contours to be mapped onto the same lamination parameter design space, and thus serve to demonstrate the effect on buckling strength of ignoring the presence of *Bending-Twisting* coupling.

New insights are provided into the relative shear buckling strength with respect to lamination parameters and lamination parameter design spaces, by way of an introduction to an accompanying article [7], which explores in detail the effect of *Bending-Twisting* coupling on compression and shear buckling strength, and is applicable to the data presented here, as well as to data for laminates with Bending-Twisting coupling only [3].

The remainder of this article is arranged as follows. Section 3 provides an overview of mechanical coupling behaviour before details of the derivation of definitive listings of *Extension-Shearing Bending-Twisting* coupled laminate configurations are presented, with up to 21 plies. Section 4 provides information on the extent of the feasible design space for *Extension-Shearing Bending-Twisting* coupled laminates, including the dominant forms of sub-sequence symmetries. *Extension-Shearing Bending-Twisting* coupled laminates arises from un-balanced and symmetric designs [6], but symmetry is shown here to be a sufficient rather than a necessary constraint. Expressions for ply orientation dependent lamination parameters are also given, together with graphical representations, which allow the available design space to be visually interrogated. The use of ply percentage mapping, as an approach to design for bending stiffness, is also discussed in the context of Quasi-Homogeneous laminates. Section 5 describes the association between ply percentages and shear buckling strength for simply supported plates, through a similar mapping procedure. Classical Garland curves are then presented in a form that permits an assessment of the bounds on the shear buckling strength of *Extension-Shearing Bending-Twisting* coupled laminates across a range of aspect ratios. Finally, conclusions are drawn in Section 6.

## 2. Mechanical coupling

Laminated composite materials are characterized in terms of their response to mechanical (and/or thermal) loading, which is associated with a description of the coupling behaviour, unique to this type of material. The coupling behaviour relevant to this study is presented in Table 1, together with the others in this series of related articles. All share the common feature that couplings between in-plane (i.e., extension or membrane) and out-of-plane (i.e., bending or flexure) responses, hence thermal warping distortions, are eliminated by virtue of the fact that $B_{ij} \neq 0$ in Eq. (1). However, these laminates possess coupling between in-plane shearing and extension when $A_{xs} = A_{ys} \neq 0$, and bending and twisting when $D_{xs} = D_{ys} \neq 0$.

$$\begin{Bmatrix} N_x \\ N_y \\ N_s \end{Bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{xy} & A_{yy} & A_{ys} \\ A_{xs} & A_{ys} & A_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{Bmatrix} + \begin{bmatrix} B_{xx} & B_{xy} & B_{xs} \\ B_{xy} & B_{yy} & B_{ys} \\ B_{xs} & B_{ys} & B_{ss} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{Bmatrix}$$
$$\begin{Bmatrix} M_x \\ M_y \\ M_s \end{Bmatrix} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xs} \\ B_{xy} & B_{yy} & B_{ys} \\ B_{xs} & B_{ys} & B_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{Bmatrix} + \begin{bmatrix} D_{xx} & D_{xy} & D_{xs} \\ D_{xy} & D_{yy} & D_{ys} \\ D_{xs} & D_{ys} & D_{ss} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{Bmatrix} \quad (1)$$

whilst Eq. (1) describes the well-known **ABD** relation from classical laminate plate theory, it is more often expressed using compact notation:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{Bmatrix} \quad (2)$$

The coupling behaviour, which is dependent on the form of the elements in each of the extensional [**A**], coupling [**B**] and bending [**D**] stiffness matrices, is now described by an extended subscript notation, defined previously by the Engineering Sciences Data Unit, or ESDU [8] and subsequently augmented for the purposes of this series of articles. Hence, laminates with coupling between Extension and Shearing, and Bending and Twisting, are referred to by the designation $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$, signifying that the elements of the extensional stiffness matrix [**A**] are finite, i.e.:

$$\begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{xy} & A_{yy} & A_{ys} \\ A_{xs} & A_{ys} & A_{ss} \end{bmatrix} \quad (3)$$