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Spectrally formulated one-dimensional element for analysis of wave propagation in pretwisted anisotropic strips

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ABSTRACT

A spectrally formulated one-dimensional element is developed to study the wave propagation in pretwisted anisotropic strips. The element is based on linear sectional analysis of pretwisted anisotropic strips with small pretwist. Firstly, the governing equations of the strip are obtained through dimensional reduction of laminated shell theory to a linear one dimensional (1-D) theory using the variational asymptotic method (VAM). Next, an exact dynamic stiffness matrix is derived in the frequency-wavenumber domain using spectral finite element (SFE) method. In SFE formulation the mass distribution is modeled exactly and as a result a single element is sufficient to capture the exact frequency response of a regular structure. For numerical validation of the proposed model, the natural frequencies of the strip are compared to the modal behavior of pretwisted composite strips available in literature. The model is used to predict wave responses due to modulated sinusoidal input. The different wave modes present, namely axial, flexural, and torsional modes are seen and their velocities are compared to that obtained from the dispersion plot. This proposed modeling strategy is a first step towards the development of a comprehensive model to do structural health monitoring of structures idealized as pretwisted strips.
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1. Introduction

Pretwisted anisotropic strips are waveguides in which the axial, flexure and torsional modes of deformation are coupled. Dispersion analysis of such a structural member serves at least three purposes: (a) benchmarking and validation of one-dimensional theories; (b) investigation of dynamic phenomenon resulting from transient loading leading to high frequency excitation; and (c) structural health monitoring (SHM) for detecting and assessing damages. A typical example where dispersion analysis of pretwisted anisotropic strips, based on a one-dimensional (1-D) model, can be beneficial is in the SHM of flexbeams used in hingeless and bearingless rotor systems. In helicopter rotor system (main and tail rotor), the flexbeams are used to accommodate the centrifugal force along with flapping, lead-lag, and torsional motions. This allows for the design of a rotor system that is independent of bearings making the design bearing-less [1]. The design of flexbeams and flexbeam-like structures found in other applications is challenging owing to the non-linearity that arises because of the large displacements and moderate rotations. This necessitates the need for the development of a 1-D theory that can accurately accommodate these deformation modes. Such a model necessarily has to provide a reasonable representation for the dispersion relationships for accurately capturing the elastic behavior of the structure [2,3]. Additionally, mission and environment under which the helicopter operates may demand understanding of high frequency response and SHM of the flexbeam. This and other engineering applications have motivated researchers to focus on modeling and understanding a more generic pretwisted anisotropic strips.

Detailed literature reviews on modeling thin-walled and thick-walled beams, keeping their application to helicopter rotors in focus, are presented in [4,5]. Challenges, especially within the context of rotor systems, in modeling such structures include assessment and determining the relative importance of physical characteristics like out of plane warping, large displacements and moderate rotations, transverse shear in case of thick cross-sections etc. on their overall deformation behavior. These complex deformation characteristics of the structure invalidates the use of 'classical' beam modeling approaches. As described by Friedmann [1], composite blade theories can be categorized into three groups: (a) variational asymptotic method (VAM) based models [6–8]; (b) theories based on ad hoc separation of the original three-dimensional (3-D) problem into a nonlinear 1-D and two-dimensional (2-D) linear cross-sectional problem [9]; and (c) thin

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walled beam models for single or two-cell composite cross-sections and 1-D theory based on moderate deflections [10]. The current state-of-the-art refined theories for beams is reviewed in detail by Carrera et al. [11]. Here, different models are grouped under the broad categories of those involving: (a) shear correction factors [12,13]; (b) displacement field and amplitude function defined over the beam, typically knows as the generalized beam theories [14,15]; (c) warping functions and Saint–Venant model [16–18]; (d) variational asymptotic method [6–8]; and (e) hierarchical framework like the Carrera Unified Formulation(CUF) [19,20]. In the CUF framework 3D problems are reduced to 2D or 1D problems in a unified manner by expanding the unknown variables of interest, like the displacement field. This method, by design, naturally overcomes the Poisson's and shear locking issues.

In the present work, VAM based theoretical framework is adopted to model the thin pretwisted strip. VAM was first proposed by Berdichevsky [21] and it has been shown to be capable of developing refined theories that are asymptotically correct even in the long-wave range and high frequency vibrations of elastic plates close to their cut-off frequencies [22]. Models based on VAM framework are capable of handling both spanwise and cross-sectional nonlinearities [23–25]. In [23] the nonlinear cross-sectional analysis was carried out to model trapeze effect in thin pretwisted strips; while, the focus was on modeling Brazier effect in [24]. Trapeze effect is especially important in thin rotating pretwisted structures. More recently, Jiang and Yu [25] have carried out dimensional reduction from 3D to nonlinear 1D analysis and 2D cross-sectional analysis by incorporating both geometrical and material nonlinearities. As a first step in attempting to study wave propagation in thin pretwisted anisotropic strip, a linearized version of the VAM based cross-sectional analysis described in [23] is adopted in the present work to determine its governing equations of motion.

Owing to the importance of pretwisted anisotropic structures considerable work has been reported on vibration and dynamic behavior of these structures, in addition to the extensive work report on static analysis. Rosard [26], Troesch et al. [27], Diprima and Handelman [28] investigated the vibration of pretwisted beams - a detailed review can be found in [29]. Subsequent applications to blade and coupling vibration were investigated by Carnegie [30,31]. Gupta and Rao [32], Sisto and Chang [33], Yardimoglu and Yildirim [34] developed the finite elements (FE) to study vibration problems of pretwisted beams or blades. Kapania and Raciti [35] have reported the advances made in the analysis of laminated beams and plates on their vibration and wave propagation behavior. An interesting observation from their review is the tendency of the laminated structures to become stiffer or weaker under certain coupling scenarios between the axial, flexure and torsional deformation modes. More recently, Filiz et al. [36] developed a spectral-Tchebychev technique for the solution of 3-D unconstrained pretwisted beams with primary focus on determining the natural frequency and mode shapes for various cross-sections. Even though much work has been done on vibration and dynamic behavior of pretwisted strips, not much work has been done on wave propagation, in particular.

Conventional FE analysis of dispersion characteristics of pretwisted anisotropic strips is challenging computationally. This takes us to the realm of frequency domain analysis and in particular, spectral analysis. This spectral analysis is used to construct a frequency domain based matrix methodology by Doyle [37], which is called spectral finite element (SFE) method. The basic steps in the development of SFE formulation begins with the transformation of the governing wave equations from time domain to frequency domain using discrete Fourier transform. This process converts the governing coupled partial differential equations (PDEs) into a set of ordinary differential equations (ODEs) in the spatial domain which can be solved exactly. The exact solutions of the ODEs results in a complex shape function matrix as a linear superposition of all the wave modes. Following the conventional FE method, the complex dynamic stiffness matrix is then formed which is exact. This makes the proposed SFE an efficient model suitable for use within the framework of automated FE method.

SFE has been widely used for study of vibration and wave propagation in 1-D and 2-D structures [38] including composite beam and laminates, functionally graded structures. In similar lines, SFE formulation of flexure-shear coupled wave propagation in delaminated multi-layer composite beam was presented by Palacz et al. [39]. Apart from the work reported on SFE formulation of anisotropic and inhomogeneous structures, Vinod et. al. [40] adopted SFE method for wave propagation analysis of tapered rotating beam. Lee et. al. [41,42] presented implementation of SFE for modeling of smart composite beams with piezoelectric layers. However, SFE formulation for analysis of pretwisted anisotropic strip is not reported in literature to the best of authors knowledge. As mentioned earlier, the use of SFE method in conjunction with VAM will allow accurate and computationally efficient framework for vibration and wave propagation studies of pretwisted anisotropic strip. This has immense potential for application to wave based structural health monitoring of such structures.

The paper is organized as follows. Section 2 presents the derivation of the kinematics and the governing differential equations adapting the VAM based model proposed in [23]. Following this, Section 3 presents the SFE formulation from the governing equations. The numerical results are discussed in Section 4. The paper ends with important conclusions and summary of the work in Section 5.

2. Pretwisted anisotropic strip model

Modeling the thin pretwisted anisotropic strip (see Fig. 1) is based on the methodology proposed by Hodges et al. [23], which uses the mathematical framework of VAM. In this section salient features of the model proposed in [23] are briefly described for completeness. The 1-D model for the thin strip is derived by reducing the actual 3-D problem into a linear cross-sectional problem and a problem along the longitudinal axis of the strip. Within the framework of VAM (see Fig. 2), this is achieved by writing the

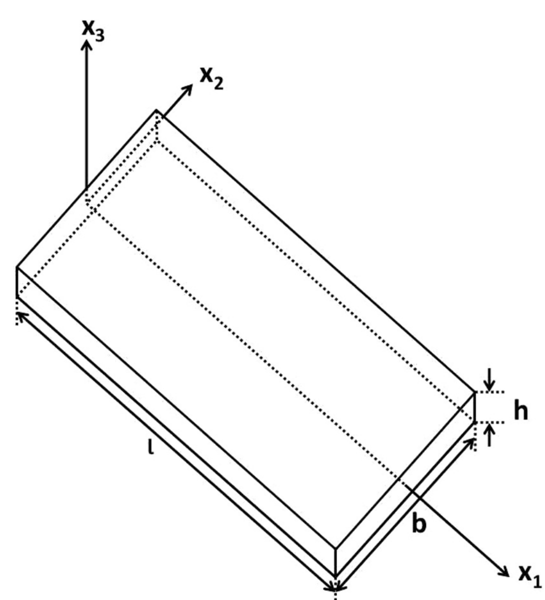

Fig. 1. Schematic representing the geometric configuration of a thin strip with a small pretwist and the coordinate system.

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