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Vibro-acoustic optimisation of sandwich panels using the wave/finite element method

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ABSTRACT

This paper investigates the use of a wave-based method in the framework of structural optimisation of composite panels involving advanced components. The wave/finite element method (WFEM) is used to evaluate the influence of a core's geometry on the transition frequency of a sandwich panel involving composite skins. This transition occurs is a sandwich panel when the transverse shear stiffness has a significant influence on the flexural motion, compared to the bending stiffness. It follows that the modal density and the acoustic radiation will considerably increase above this frequency. The periodic waveguide is modelled at the mesoscopic scale using a 3D finite element model of the unit-cell. Therefore this method does not require an homogenisation of the core based on Gibson and Ashby formulations to provide the wave dispersion characteristics. Although the cellular cores compared in this study share the same mass-to-stiffness ratio, a significant alteration of the transition frequency and modal density can be observed compared to honeycomb cores. A periodic octagonal core is designed, providing up to 70% increase of the transition frequency and a significant reduction of the modal density.

1. Introduction

Sandwich panels are extensively used in automotive, naval and aerospace industry. These structures have high stiffness-to-weight ratios and are usually made of a moderately thick, lightweight core surrounded by glass or carbon-fibre composite skins. The core can be made of polymer foam or more complex structures involving aluminium or resin honeycomb constructions. Although the material and geometrical properties of the panel are usually designed to provide specific stiffness and density characteristics, structural optimisation also concerns acoustic radiation efficiency in order to improve the acoustic comfort. Therefore, there is an increasing need for reliable optimisation tools for design engineers, providing fast vibro-acoustic evaluation of large-scaled structures involving composite components.

In this context, the knowledge of accurate wave dispersion characteristics in two laminated orthotropic skins connected by 3D cellular honeycomb core is a key information for the prediction of the acoustic transmission parameters. Numerical methods for analysing the vibrational behaviour of complex composite or periodic panels in a broadband frequency range were extensively investigated in the last decade. Honeycomb sandwich panels were usually modelled using classical laminate plate (CPLT) of Reissner–Mindlin (FSDT) theories involving three homogeneous layers. An extensive number of analytical formula for analysing wave dispersion characteristics in two-dimensional waveguides can be found in the literature (see Reddy [1]), starting with the asymptotic model for analysing wave dispersion in thick symmetric sandwich panels developed by Kurtze and Watters [2], then later in the classic work of Wilkinson [3] and Erickson [4]. The prediction of wave propagation characteristics of structurally advanced structures using analytical models has been a subject of intense research over the past years. Renji et al. [5] introduced a new analytical formulation providing a core transverse shear stiffness and bending matrices in both directions for orthotropic honeycomb sandwich panels. Ghinet and Atalla [6] also used an analytical model based on Discrete Laminate Theory (DLT) to compute the dispersion characteristics of layered structures and obtain their acoustic parameters.

Recently, Guillaumie [7] proposed an analytical solution for the eigenmodes and modal densities of symmetric sandwich panels involving composite skins and honeycomb core. This model was found to be effective in the low- and medium-frequency range for moderately thick sandwich structures and was confirmed by experiments. Florence and Renji [8] also derived explicit solutions for the modal density of composite cylinders.

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However, these analytical solutions require a homogenisation of the hexagonal cells of the core based on Gibson and Ashby formulations [9]. These analytical approaches therefore suffer some drawbacks in the medium-frequency range, since they cannot handle arbitrarily shaped 3D cells for the core. Besides, these models are often inaccurate when compared to experimental characterizations in higher frequency range, especially when dealing with complex industrial products involving thick, multi-layered constructions made of anisotropic glass or carbon fibres skins.

Therefore, numerical methods were recently developed to perform wave analysis in complex periodic waveguides defined using classical finite element packages (FEM). Among others, the wave finite element method (WFEM) [10,11] has been successfully applied to predict the radiation efficiency of numerous thick layered structures. Nevertheless, these models involve homogeneous orthotropic layers for the honeycomb construction, and may become inaccurate when the wavelengths are reaching the periodic cells dimensions. Although the design optimisation of honeycomb cells has received a lot of attention recently [12,13], the dispersion characteristics of sandwich panels involving realistic composite skins and arbitrarily shaped cores was not investigated to the author's knowledge.

This paper focuses on the influence of the meso-scale parameters of the core's construction on the vibro-acoustic response of the sandwich panel. A special interest is given to the modal density, a typical vibro-acoustic parameter and the transition frequency, associated with the passage from a behaviour where the flexural wave is ruled by the skins stiffness to a behaviour where the core's transverse shear parameters governs the flexural motion. First, finite element models are developed for the periodic cells. The WFEM, which combines FEM and the Periodic Structures Theory (PST) is applied to evaluate the wave dispersion characteristics. Since numerous degrees of freedom are usually involved to ensure accuracy in a broadband frequency range, reduced order modelling (ROM) was recently investigated for periodic [14,15] and large-scaled [16] waveguides. A hybrid wave-mode ROM is therefore applied to provide suitable computational efficiency. When dealing wit hexagonal cores, the equivalent orthotropic material can be described using Gibson and Ashby formulations, and the analytical expression of the wavenumber can be derived for the symmetric sandwich. The possibility to use a regression on the wavenumber function is therefore examined for various core's configurations, in order to derive an analytical expression of the transition frequency from the equivalent bending and shear parameters. Finally, the effects of several geometrical parameters of the core on the modal density and the transition frequency are investigated.

2. Wave propagation in 2D periodic structures

2.1. The wave finite element method

Numerous formulations of the WFEM were given in the literature. The method is briefly reviewed hereby. Consider a periodic–cell of a 2D waveguide of dimensions d_x and d_y in x and y directions (see Fig. 1), modelled using a FE software. The mass $\mathbb{M}$, stiffness $\mathbb{K}$ and damping $\mathbb{C}$ matrices can be extracted and the dynamic stiffness matrix is written at the circular frequency $\omega : \mathbb{D} = \mathbb{K} + j\omega\mathbb{C} - \omega^2\mathbb{M}$. The degrees of freedom $\mathbf{q}$ (DOF) on the edges, corner and centre of the cell can be reordered as follows:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_I^T & \mathbf{q}_B^T & \mathbf{q}_T^T & \mathbf{q}_L^T & \mathbf{q}_R^T & \mathbf{q}_1^T & \mathbf{q}_2^T & \mathbf{q}_3^T & \mathbf{q}_4^T \end{bmatrix}^T. \quad (1)$$

It should be noted that apart from the mesh compatibility between the opposite edges of the unit-cell, the discretization is arbitrary inside the periodic element. Then using the periodicity

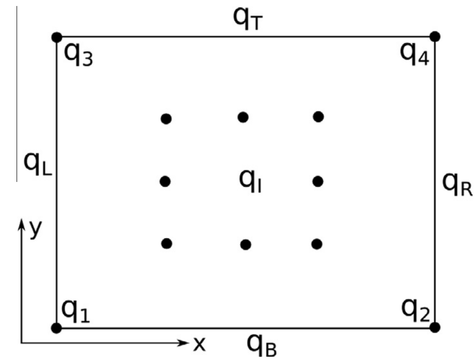

Fig. 1. Top-view of the modelled periodic element with its edge (B, T, R, L), corner (1, 2, 3, 4) and inner (I) degrees of freedom.

relations and Bloch's theorem [17], a reduced state vector can be defined using the propagation constants λ_x and λ_y in x- and y-directions:

$$\begin{pmatrix} \mathbf{q}_I \\ \mathbf{q}_B \\ \mathbf{q}_T \\ \mathbf{q}_L \\ \mathbf{q}_R \\ \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \\ \mathbf{q}_4 \end{pmatrix} = \begin{pmatrix} \mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & \mathbf{I}\lambda_y & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{I}\lambda_x & 0 \\ 0 & 0 & 0 & \mathbf{I} \\ 0 & 0 & 0 & \mathbf{I}\lambda_x \\ 0 & 0 & 0 & \mathbf{I}\lambda_y \\ 0 & 0 & 0 & \mathbf{I}\lambda_x\lambda_y \end{pmatrix} \begin{pmatrix} \mathbf{q}_I \\ \mathbf{q}_B \\ \mathbf{q}_L \\ \mathbf{q}_1 \end{pmatrix} = \mathbf{R}\mathbf{q}'. \quad (2)$$

Then, denoting k_x, k_y the wavenumbers in x and y directions, the propagation constants $\lambda_x = e^{-jk_x d_x}$ and $\lambda_y = e^{-jk_y d_y}$ are the solutions of the problem:

$$\mathbf{R}^T(\lambda_x, \lambda_y)\mathbb{D}\mathbf{R}(\lambda_x, \lambda_y)\mathbf{q}' = \mathbf{0}. \quad (3)$$

These solutions describe the propagating and evanescent waves in the periodic waveguide. Each solution (k_x, k_y, ω) is associated with an eigenvector Ψ describing the deformed shape of a given wave. The propagating and evanescent waves can be discriminated from the imaginary parts, since $(\Im(k_x), \Im(k_y))$ describes wave attenuation in x- and y-direction, respectively. In dissipative waveguides, all the waves are decaying and a customised selection procedure, usually based on the value of $\frac{|\Im(k)|}{\Re(k)}$ can be employed.

2.2. Resolution and post-processing

It should be noted that Eq. (3) is a transcendental eigenvalue problem whose resolution is not straightforward when internal DOFs are involved. In order to turn this problem into a classical eigenvalue problem, two parameters are usually given (the circular frequency ω and the wavenumber in one direction: k_x or k_y). The directional wavenumber $k(\theta, \omega)$ can be obtained from the discrete $(k_x(\omega), k_y(\omega))$ solutions. An accurate description of the 2D dispersion curves (also referred as *k-space*) hence requires an important number of resolutions of Eq. (3).

In order to enable fast design optimisation, through broadband wave analyses of finely meshed periodic honeycomb constructions, a reduced formulation of the WFEM (see [18]) is used in the numerical applications. It employs a projection of the state vector $\mathbf{q}'$ on solution subset involving a reduced number of wave solutions and component modes of the periodic cell.

The flexural waves can then be discriminated using a wave-matching algorithm. At each frequency step, the wave shape

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