Composite Structures xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Maximization of the fundamental frequency of plates and cylinders

D. Trias [*,1], P. Maimí, N. Blanco

AMADE, Dept. of Mechanical Engineering and Industrial Construction, University of Girona, Campus Montilivi s/n, E-17071 Girona, Spain

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Optimization
Lamination parameters
Modal analysis

ABSTRACT

This work deals with the maximization of the fundamental frequency of laminated plates and cylinders by finding the optimal stacking sequence for symmetric and balanced laminates with neglectable bending–twisting and torsion–curvature couplings are considered. Instead of using the angle at each ply the so-called lamination parameters are used as design variables. Results for the plate problem are compared to those using analytical expressions found in the literature. The cylinder problem is solved for a large number of thickness and radius-to-length ratio combinations. Values of the optimal stacking sequence are given considering the angle can take any value between 0° and 90° or the case when the angle can only take a discrete set of values.

1. Introduction

Composite structures are specially useful in aeronautic applications where the need for materials with high stiffness/weight ratio is crucial. Aeronautic structures generally require the fundamental frequency to be higher than those of the excitations. If this requirement is practically unattainable, then the fundamental frequency is required to lie between two excitation frequencies. In the first case, structures with a maximum fundamental frequency are desired and in the second structures are optimized so the fundamental frequency is precisely located between two known excitation frequencies [1]. In other cases, generally if many structural requirements have to be satisfied, the natural frequency is only treated as a constraint together with some failure index or the buckling load, while the total mass is minimized [2,3]. In some cases, the maximization of the fundamental frequency can be coupled with the maximization of the buckling load in multi-objective optimization environments [4].

In the case of aiming to maximize the fundamental frequency, stacking sequence optimization is a useful method which can be carried out considering different types of design variables. The most natural and intuitive option is to take the angle layer as a design variable and to have as many design variables as layers or half the number of layers if only symmetric laminates are considered. This approach is generally named layerwise optimization and has been extensively applied [1,5–8]. This may lead to a large number of design variables and the consequent computational costs, so other options are also considered. Rao and Singh [2] predefined some available angles and used the thickness associated to each angle as a design variable. Other researchers have proposed some codification or parametrization procedures for discrete values of the layer angle [9,10]. The resulting design variables, allow to use gradient-based optimization methods which require continuous variables. In this direction, the use of lamination parameters [11] provides a robust method which converts any stacking sequence to a set of continuous variables between −1 and 1. Some of these variables are linked by inequalities which can be easily integrated in the optimization procedure as constraints. Lamination parameters are used in this work and so they are explained with some more detail in Section 2.2.

Regarding the solution methods for stacking sequence optimization, although gradient based methods offer a robust mathematical background they are not able to deal with discontinuities and discrete variables. For this reason, some well known heuristic solution methods have been used successfully in the literature; to mention just a few: Particle Swarm Optimization [8], Genetic Algorithms [12–14] and Ant Colony Optimization [15,16]. The review written by Ghiasi and co-workers [17] summarizes the different approaches for the choice of the design variables and the main solution methods used for stacking sequence optimization.

One of the first scientific works devoted to the maximization of the fundamental frequencies of composite material components was written by Bert [18] and it was devoted essentially to plates. The results of his work will be used and compared with the result of the present work in the following sections.

* Corresponding author. Tel.: +34 972418149; fax: +34 972418098.
E-mail address: dani.trias@udg.edu (D. Trias).
[1] Serra Hunter Fellow.

http://dx.doi.org/10.1016/j.compstruct.2015.08.034
0263-8223/

Rao and Singh [2] tackled the minimization of weight considering the natural frequency as a constraints together with buckling and failure criteria and also the maximization of the fundamental frequency considering buckling load and failure criteria as constraints.

As a structure of common interest in different industries, the optimization of the stacking sequence of composite cylinders for maximum natural frequency has been addressed frequently. Hu and Tsai [19] considered the maximization of the fundamental frequency in cylinders and cylinders with a cutout. They solved the optimization problem using the Gold Section method. Koide and Luersen [16] also considered the maximization cylinders and cylinders with a cutout using Ant Colony Optimization. The angle at each layer is forced to be 0°, $\pm 45^\circ$ or 90°. Shakeri et al. [20] solved the problem restricting the angle layer to some values and using Genetic Algorithms. Their attention is more focused in the algorithm itself than in the composite structure.

In opposition to what happens with plates, the fundamental vibration mode of curved shells is not restricted to $(m, n) = (1, 1)$ but generally takes other values, depending on the curvatures. This generally adds more complexity to the optimization process, since several vibration modes have to be monitorized. Narita and Zhao [5] found the laminates which give the maximum fundamental frequencies of doubly curved shallow shells, under different aspect ratios, using gradient based optimization together with Lagrange multipliers to consider the constraints. They reported difficulties when the functions of the frequency of the three involved vibration modes (1,1), (1,2) and (2,1) had intersection points and used an specific bisection method to find the optimum. However, when the number of modes involved increases, so does the complexity of finding the vibration mode which leads to the fundamental frequency and consequently, the optimization problem becomes much more difficult to solve.

To the authors' knowledge lamination parameters have never been used to obtain the optimal stacking sequence for fundamental frequency maximization and the effect of different types of laminate regarding the angles involved on the maximal fundamental frequency has not been reported. This work deals with the maximization of the fundamental frequency of laminated plates and cylinders using lamination parameters as design variables. Section 2.2 summarizes the theory behind the use of lamination parameters. Next, the main equations regarding modal analysis of composite plates and cylinders are exposed in Section 3. The proposed optimization problem is described in Section 4 while results are presented and discussed in Section 5.

2. Fundamental equations

2.1. Classical Laminate Theory (CLT)

Classical Laminate Theory (CLT) assumes that the N layers which constitute the laminate are perfectly bonded, assuming compatibility of deformations between layers. Under these assumptions, the constitutive relations of the laminate can be written:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \cdot \begin{Bmatrix} \boldsymbol{\epsilon} \\ \boldsymbol{\kappa} \end{Bmatrix} \tag{1}$$

where $\mathbf{N}$ and $\mathbf{M}$ are normal force and torsion moment vectors defined respectively by $\mathbf{N}^T = \{N_x, N_y, N_z\}$ and $\mathbf{M}^T = \{M_x, M_y, M_{xy}\}$, and $\boldsymbol{\epsilon}, \boldsymbol{\kappa}$ are deformation and curvature vectors defined respectively by $\boldsymbol{\epsilon}^T = \{\epsilon_x^0, \epsilon_y^0, \gamma_{xy}\}$ and $\boldsymbol{\kappa} = \{\kappa_x, \kappa_y, \kappa_{xy}\}$. On the other hand the $\mathbf{A}, \mathbf{B}, \mathbf{D}$ matrices can be computed:

$$A_{ij} = \sum_{i=1}^{N} [\overline{Q}_{ij}]_i (z_i - z_{i-1}) \tag{2}$$

$$B_{ij} = \frac{1}{2} \sum_{i=1}^{N} [\overline{Q}_{ij}]_i (z_i^2 - z_{i-1}^2) \tag{3}$$

$$D_{ij} = \frac{1}{3} \sum_{i=1}^{N} [\overline{Q}_{ij}]_i (z_i^3 - z_{i-1}^3) \tag{4}$$

where z_i is the mean position of the ith layer with respect to the mean thickness plane of the laminate and $\overline{Q}_{ij}$ are components of the lamina reduced stiffness matrix which considers the layer angle. A common way to express the former equations [21] is:

$$\{A_{ij}, B_{ij}, D_{ij}\} = \int_{-h/2}^{h/2} \overline{Q}_{ij} \{1, z, z^2\} dz \tag{5}$$

In most of practical situations symmetric laminates are employed because they avoid the bending–extension coupling (that is, $\mathbf{B}$ in Eq. (1) is null).

Another common strategy to avoid coupling effects is the use of balanced laminates, that is, laminates which use a negative angle plie for each positive angle ply. In that case, couplings between in-plane normal forces and in plane shear deformation, and between in-plane shear forces and in-plane normal deformation are avoided ($A_{16} = A_{61} = A_{26} = A_{62} = 0$). In the case of the bending–twisting coupling and torsion moment–curvature coupling (terms D_{16}, D_{26}, D_{61} and D_{62}), they could only be avoided with uni-directional 0° laminates, cross-ply laminates or balanced and anti-symmetric laminates. However, in practical situations these couplings could be almost avoided by using stacking sequences which reduce the involved terms in the matrix to almost 0. In this paper balanced and symmetric laminates in which $D_{16} = D_{26} = D_{61} = D_{62} \approx 0$ are considered in this work.

2.2. Lamination parameters

The optimization of the stacking sequence of laminated composites implies, given the properties of a lamina, the finding of the orientation of each of the N laminae which minimizes/maximizes some objective function. This would imply N design variables which for, usual thicknesses would mean from 8 to 50 variables. Although this number is reduced in a half when using symmetric laminates, the use of lamination parameters allows to solve this type of optimization problems with a reduced set of design variables. In a general problem, for a given thickness the number of variables reduces to 12 or to 8 for symmetric laminates. In the type of laminates considered in this work (balanced and symmetric laminates in which $D_{16} = D_{26} = D_{61} = D_{62} \approx 0$) only 4 variables for a given thickness are needed. The complete theoretical background is concisely documented in [21]. Here only the essentials are summarized.

Tsai and Pagano [22] found the following magnitudes (functions of the $\mathbf{Q}$ matrix terms) to be invariant to any ply orientation:

$$\begin{aligned} U_1 &= \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}) \\ U_2 &= \frac{1}{2}(Q_{11} - Q_{22}) \\ U_3 &= \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}) \\ U_4 &= \frac{1}{8}(Q_{11} + Q_{22} + Q_{12} - 4Q_{66}) \\ U_5 &= \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \end{aligned} \tag{6}$$

where Q_{ij} are components of the lamina reduced stiffness matrix. When layers are all of the same material, these definitions can be used to express the terms of the matrices $\mathbf{A}, \mathbf{B}, \mathbf{D}$ as a function of these invariants. For instance [21]: