Composite Structures xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Application of higher order finite differences to damage localization in laminated composite plates

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Damage localization
Laminated composite plate
Finite differences
Mode shape curvatures
Optimal spatial sampling
Ritz method

ABSTRACT

The localization of damage in composite plates and other structures is often performed based on mode shape curvatures. These curvatures are usually computed using finite differences. However, finite differences spread and amplify numerical and experimental errors. A technique based on the Ritz method allows to choose an optimal spatial sampling in order to minimize this problem. In the present work we apply this technique, along with the second and fourth order finite differences to compute the mode shape curvatures. To evaluate the successfulness of the damage localizations a quality evaluator is proposed in this paper. The need for the optimal spatial sampling is verified by analyzing two locations of a wide range of damage severities. Damage localizations obtained with second and fourth order finite differences were compared and it was found out that the results are better when one uses the highest order finite difference.

1. Introduction

Methods for the detection and localization of damage in composite structures using vibration characteristics have been extensively reported in the literature [1,2]. These methods are usually based in the relationships among damage, the decrease in stiffness and the changes in vibration behavior. Although one can detect the presence of damage using natural frequencies, the localization of damage is a more challenging task. Thus, it is necessary to make use of more sensitive dynamic parameters. Due to the direct relationship of the curvature, bending moment and stiffness of beams and plates at a local level, the second order spatial derivatives of mode shapes can be used to localize damage. One of the most reliable methods, based on the differences of mode shape curvatures of undamaged and damaged beams, was proposed by Pandey et al. [3]. The computation of the curvatures is accomplished by applying the second order central finite difference formula to the measured displacements fields. Very recently, an overview and comparison of methods based on modal curvatures have been presented [4]. The performance of the methods is verified with an Euler–Bernoulli beam. Also recently, a review of similar techniques and the localization of damage in Euler–Bernoulli and Timoshenko beams was published [5]. One may also use the differences in the rotation or slope of mode shapes to localize damage, and such a method was proposed by Abdo and Hori [6]. Zhu et al. [7] applied the changes in the slope of the first mode shape to detect damage in an eight-storey numerical example and a three-storey experimental model.

Many of the methods found in the literature can lead to incorrect damage identifications due to the propagation of the measurement errors, which are always present in experimental data. Being a numerical approximation technique, the finite difference itself also originate errors in the computation of the derivatives needed and thus incorrect results are obtained. In order to minimize these problems, some improvements on the methods cited above and on similar ones, like the damage index [8] and the frequency response functions curvature [9] methods, have been reported. For instance, Maeck and De Roeck [10] suggested a preliminary smoothing in order to obtain reasonable derivatives. An adaptive eigenanalysis was proposed by Dutta and Talukdar [11] to improve the accuracy of the modal parameter evaluation. The problem of differentiating noisy data was minimized by dos Santos et al. [12] by applying a differentiation/smoothing technique. If one uses the curvature differences of the frequency response functions, the simple summation of the information at different frequencies may mask the true location of the damage. Thus, the sum of the maximum occurrences in the frequency range, normalized to the number of

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http://dx.doi.org/10.1016/j.compstruct.2015.08.011
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total occurrences is a better indicator of damage as shown in [13]. As pointed out by Lestari et al. [14], if one uses a scanning laser vibrometer, the resulting large number of measurement points also increases the quality of the measured displacement mode shapes and thus the accuracy of the mode shape curvatures. Guan and Karbhari [15] proposed a method where the derivatives are estimated using spline interpolation and numerical optimization. In order to cope with uncertainty in structural damage detection, Chandrashekhar and Ganguli [16] proposed the combination of the mode shape curvature based method and fuzzy logic. Tomaszewska [17] developed a technique combining deterministic and stochastic approaches to distinguish the false results from the true ones, using modal curvature and structural flexibility. Another approach to improve the damage identification is the use of signal processing tools such as wavelets [18]. A method based on a modified Savitzky–Golay filter and cubic splines was proposed recently by Quaranta et al. [19].

The influence of measurement and numerical errors can also be diminished by defining and selecting an optimal spatial sampling, as proposed by Sazonov and Klinkhachorn [20] and Moreno-García et al. [21]. In the work of Sazonov and Klinkhachorn [20], the optimal spatial sampling for the computation of the mode shape curvature of isotropic beams with the second order central finite difference formula is obtained based on the discretization of a number of sampling intervals. This allows the estimation of the maximum of the mode shapes fourth order derivative. In the more recent work of Moreno-García et al. [21] the optimal spatial sampling is obtained by minimization of the total error and the use of the Ritz method to obtain higher order derivatives of the modes shapes. Furthermore, the formulation is applied to laminated composite plates. The damage studied in reference [21] is described by a global reduction of the laminated stiffness and results with only a value of the damage parameter are presented. With this damage parameter one is not able to define the reduction of stiffness in a given direction and thus may not describe correctly a actual damage.

In the present work, the damage is defined by a stiffness reduction in the longitudinal direction of the laminated plate, which according to Wang et al. [22] has the most influence on the vibration characteristics. Besides this new damage model, a comprehensive study of the intensity of damage is carried out. Because the optimal spatial sampling depends on the kind of finite difference formula used to compute mode shape curvatures, this dependency is also studied and the errors of central finite differences of order two and four are compared. A comparison of the quality of damage localizations is also performed using a statistical evaluator.

2. Method

2.1. Optimal spatial sampling

The curvature of the qth mode shape of a plate rectangular in a point with coordinates (x_j, y), relative to the x direction can be computed with a central finite difference of order m, according to the following expression:

$$\frac{\partial^2 w_q(x_j,y)}{\partial x^2} \approx \frac{2!}{m! h_x^2} \sum_{i=0}^{m} P_i w_q(x_i,y) \quad (1)$$

where h_x is the spatial sampling and P_i are known coefficients, presented in Table 1 for $m = 2$ and $m = 4$. Fig. 1 shows the position of the point where the curvature is computed in relation to its neighboring points, with a spatial sampling h_x. One sees that the computation using second ($m = 2$) and fourth ($m = 4$) order finite differences involves the knowledge of the displacement in three and five points, respectively. Eq. (1) is an extension to two dimensional functions of the mathematical formulas found in [23]. A similar expression can be written for the computation of the curvature in the y direction.

Table 1
Coefficients P_i, $C^{(r)}$ and $C^{(m)}$ for the computation of mode shape curvatures, mean errors, and optimal spatial sampling.

m	j	P_0	P_1	P_2	P_3	P_4	$C^{(r)}$	$C^{(m)}$
2	1	1	−2	1			4	1/12
4	2	−1	16	−30	16	−1	64	1/90

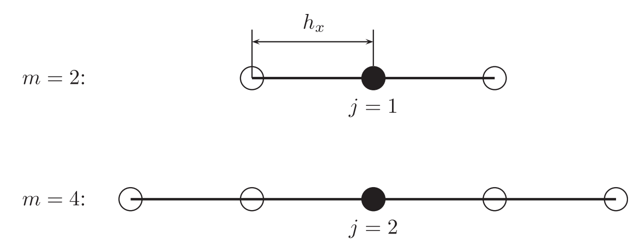

Fig. 1. Point where the curvature is computed • and its neighboring points ∘ using central finite differences of order $m = 2$ and $m = 4$ with a spatial sampling h_x.

Following the formulation presented previously in reference [21], the mean value of the round-off error in the computation of the mode shape curvature is given by:

$$\overline{E_q^{(r)}(x,y)} \simeq C^{(r)} \frac{\epsilon}{m! h_x^2} \overline{|w_q(x,y)|} \quad (2)$$

where the overbar denotes the mean value of the quantities and ϵ corresponds to the measurement accuracy.

The mean value of the intrinsic error of the finite differences is dependent on their order, such that:

$$\overline{E_q^{(m)}(x,y)} \simeq h_x^m C^{(m)} \overline{\left|\frac{\partial^{m+2} w_q(x,y)}{\partial x^{m+2}}\right|} \quad (3)$$

Thus, for the central finite differences of order $m = 2$ and $m = 4$, the mean errors are, respectively:

$$\overline{E_q^{(m)}(x,y)} \simeq h_x^2 C^{(m)} \overline{\left|\frac{\partial^4 w_q(x,y)}{\partial x^4}\right|} \quad (4)$$

and

$$\overline{E_q^{(m)}(x,y)} \simeq h_x^4 C^{(m)} \overline{\left|\frac{\partial^6 w_q(x,y)}{\partial x^6}\right|} \quad (5)$$

One sees in Eqs. (2) and (3) that the mean round-off error decreases with the spatial sampling h_x, whereas the mean error of the finite differences increases with the spatial sampling. A typical variation of the total mean error, which is the sum of the errors discusses above, for a given measurement accuracy ϵ, is presented in Fig. 2. The plotted curves have left and right sides, separated by a point with the minimum value of the total error. The left sides of both curves are parallel and correspond to the round-off error given by Eq. (2). The round-off error of the fourth order finite difference is higher than the round-off error of the second order finite difference. This difference is only due to the different values of $C^{(r)}$ and m in the definition of the round-off error according to Eq. (2). The right side of the error curve of the fourth order finite difference has a slope that is higher than the right side of the error curve of the second order finite difference. This means that the error increases more rapidly with the spatial sampling for the highest order finite difference. The reason for this, as one sees in Eq. (5), is that the error is a function of the sixth derivative and not the

Please cite this article in press as: Moreno-García P et al. Application of higher order finite differences to damage localization in laminated composite plates. Compos Struct (2015), http://dx.doi.org/10.1016/j.compstruct.2015.08.011