Computers and Geotechnics 88 (2017) 46–60

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Particulate material fabric characterization by rotational haar wavelet transform

Junxing Zheng, Roman D. Hryciw *

Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI 48109-2125, USA

ARTICLE INFO

Article history:
Received 20 December 2016
Received in revised form 13 February 2017
Accepted 28 February 2017

Keywords:
Soil fabric
Particle shape
Image analysis
Wavelet transform
Sand

ABSTRACT

A Rotational Haar Wavelet Transform (RHWT) method is developed to characterize the fabric of particulate assemblies from two-dimensional images. A Maximum Energy Ratio Ψ reveals the fabric direction and its intensity. The method is implemented on 12 sand and 3 rice specimens of various shapes. It was shown that Ψ may be expressed in terms of a material's aspect ratio and relative density. A material fabric classification system based on Ψ is proposed. The parameter also defines the fabric tensor for cross anisotropic material. Scanning electron microscope images of kaolinite clay and several rock images are also analyzed.

1. Introduction

Because of depositional processes, fabric cross anisotropy (or transverse isotropy) is commonly encountered in earth materials [1–4]. Within the depositional plane, the soil particles' long axes point in random directions thus developing an isotropic fabric within that plane. This contrasts with the fabric established in the direction of deposition. The fabric anisotropy endows earth materials with macroscopic properties including strength, dilatancy, permeability and compressibility that vary with direction. This distinguishes soil from other civil engineering materials such as steel and concrete.

Laboratory tests for investigating soil behavior in different fabric directions were first performed on clays, perhaps because it is relatively easy to prepare anisotropic clay specimens by sampling intact blocks in different directions. The early works of Hansen and Gibson [5], Mitchell [6], Jakobson [7], Lo [8], and Duncan and Seed [9] all showed that the shear strength of clays varies with the angle between the major principal stress and the deposition plane. Following these early studies, the influence of fabric anisotropy on strength, permeability, and compressibility of clay were systematically studied by Lo and Milligan [10], Saada [11], Lo and Morin [12], Saada and Ou [13], Prevost [14], Yong and Silvestri [15], Graham and Houlsby [16], Kirgard and Lade [2], Dewhurst et al. [17], Kurukulasuriya et al. [18], Nishimura et al. [19], Anantanasakul et al. [20] and many others.

Laboratory studies of the macro behavior of sands at different fabric directions began in the 1970s. It is somewhat difficult to prepare anisotropic sand specimens with varying fabric directions. One way is to freeze a wet specimen or impregnate an air-pluviated specimen with a resin to maintain its fabric and then trim the specimen in different orientations [21,22]. Another way is by using devices that incline a test vessel at various angles during air pluviation [23–25]. Following one of these specimen preparation procedures, various laboratory tests have been performed to study the influence of fabric anisotropy on macro behavior of sands. Studies have included plane strain compression tests by Oda et al. [22], Oda [26], and Tatsuoka et al. [23]; direct shear tests by Azami et al. [27], Guo [24], and Tong et al. [25]; triaxial tests by Oda [21], Arthur and Menzies [28], Arthur and Phillips [29], Wong and Arthur [30], Arthur et al. [31], Oda et al. [22], Lam and Tatsuoka [32], and Ochiai and Lade [33]; torsion shear tests by Rodriguez and Lade [34] and Yang et al. [35]. The influence of fabric anisotropy on bearing capacity and seismic response of foundations was studied by Meyerhof [36], and Oda and Koishikawa [37]. Besides physical laboratory tests, the effects of fabric anisotropy were also observed in Discrete Element Method (DEM) studies such as in the works of Hosseininia [38], Ng [39], Zhao and Guo [40], Fu and Dafalias [41]. Yimsiri and Soga [42]. Based on these laboratory and numerical observations, constitutive models that accounted for fabric anisotropy were developed by Pietruszczak and Mroz [43,44], Li and Dafalias [45], Dafalias et al. [46], Lade

---

* Corresponding author.
E-mail addresses: junxing@umich.edu (J. Zheng), romanh@umich.edu (R.D. Hryciw).

http://dx.doi.org/10.1016/j.compgeo.2017.02.021

[47,48], Schweiger [49], Liu and Indraratna [50], Yao and Kong [51], Kong et al. [52], and Gao and Zhao [53].

Despite an abundance of evidence regarding the importance of fabric anisotropy on mechanical soil behavior, fabric anisotropy is rarely considered in routine geotechnical design and analysis. The primary reason is the lack of an easy, effective and accurate method to quantify it. The fabric anisotropy should be quantified from two aspects, the primary fabric direction and the degree of anisotropy. These two parameters can alternatively be computed based on the orientations of non-spherical particle long axes, from the orientations of interparticle contacts and, or the orientations of non-spherical voids (Oda and Nakayama [1]). The fabric computed from the orientations of non-spherical particle long axes is the focus of this study. This is the most frequently used fabric definition because it can be readily determined by visual observation. For example, Oda [21] impregnated resin into sand specimens then cut them into thin sections of thickness between 0.04 and 0.07 mm. He manually measured the long axis orientations of 200 particles to determine fabric orientation. Oda's [21] method was later augmented by digital image processing techniques. Yang et al. [56] cut specimens at different locations (but not thin sections). They polished the surfaces before capturing images. Fonseca et al. [54,55] captured the section images of specimens using micro-computed tomography. Instead of manually counting particle orientations, Yang et al. [56] and Fonseca et al. [54,55] used image thresholding techniques to identify individual particles and compute the orientation distributions of particle long axes. Requisite to this method is that sand particles must have a different color from the surrounding resin. The greatest challenge is to computationally determine the boundaries of contacting particles so that their orientations could be assessed individually.

Existing fabric and anisotropy quantification methods are all based on statistical distributions of individual particles' long axes. Although it is well known that elongated particles will develop stronger fabric anisotropy than spherical particles, particle shape is not typically considered. Guo [24] showed that direct shear strength systematically increases as particle elongation increases from perfectly round glass beads to sub-rounded Ottawa sand to angular crushed limestone. Similar observations were made by Oda [26], Lade[48], and Tong et al. [25]. Therefore, complete fabric characterization should logically include particle elongation.

To meet the challenge of simultaneously characterizing and quantifying the fabric and shape of particles from images, a new *Rotational Haar Wavelet Transform* (RHWT) technique is proposed in this paper. Instead of identifying individual particles, the RHWT analyzes grayscale distributions and their changes across an image. This allows the method to be utilized on images of contacting particle assemblies; although, it could just as well be used on individual particles. Images of clays and rock will also be analyzed to demonstrate the method's ability to characterize fabric in other earth materials. Finally, in recognition of the wide use of fabric tensors in soil mechanics and constitutive modeling, a method for determining the fabric tensor using the Rotational Haar Wavelet Transform will be presented.

2. Haar Wavelet Transform

The Haar Wavelet Transform (HWT) has been used extensively for digital image compression, denoising, edge detection, feature extraction, texture analysis and image segmentation. The HWT is also not new in geotechnical engineering. It has been used to determine particle size distributions by Shin and Hryciw [57], Hryciw et al. [58] and Ohm and Hryciw [59], and for analysis of surface roughness by Chandan et al. [60]. The rigorous mathematical formulation of the HWT is detailed in many textbooks and will not be repeated here. However, the authors will illustrate the essence of the HWT through a simple example that will illustrate why the HWT is ideally suited for analysis of fabric anisotropy.

The example is the idealized 8 × 8 pixel grayscale image A_0 shown in Fig. 1. The larger grayscale values shown correspond to brighter areas in the image. They change only in the horizontal direction while staying constant vertically. This reflects a strongly vertical fabric.

The HWT first divides the image into 2 × 2 subareas as shown in Fig. 2(a). The location of each 2 × 2 subarea can be represented by $A_0(i, j)$ where i and j range from 1 to 4.

Assuming the four numbers in an (i, j) region are:

$$A_0(i,j) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{1}$$

the HWT computes four values $A_1(i, j)$, $H_1(i, j)$, $V_1(i, j)$, and $D_1(i, j)$. The $A_1(1, 1)$ is twice the average of the four numbers:

$$A_1(i,j) = \frac{a+b+c+d}{2} \tag{2}$$

The $H_1(i, j)$ is the average difference between two columns:

$$H_1(i,j) = \frac{(a+c)-(b+d)}{2} \tag{3}$$

The $V_1(i, j)$ is the average difference between two rows:

$$V_1(i,j) = \frac{(a+b)-(c+d)}{2} \tag{4}$$

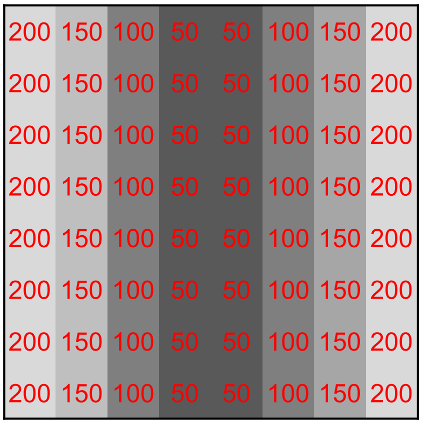
The $D_1(i, j)$ is the average difference between two diagonals:

$$D_1(i,j) = \frac{(a+d)-(b+c)}{2} \tag{5}$$

For our example, the computed results for the first subarea $A_0(1, 1)$ are: $A_1(1, 1) = 350$, $H_1(1, 1) = 50$, $V_1(1, 1) = 0$, and $D_1(1, 1) = 0$. These computations are repeated for all 16 subareas which yield the 4 × 4 matrices A_1, H_1, V_1, and D_1 shown in Fig. 2. Only A_1, H_1, and V_1 are useful for fabric characterization. The matrix A_1 (which can be thought of as a 4 × 4 image) is essentially a downscaling of A_0 by a factor of 2. The matrix H_1 quantifies the grayscale change in the horizontal direction while V_1 quantifies the grayscale change in the vertical direction. Since there is no grayscale change in the vertical direction V_1 is a zero matrix.

The *energy* (E) is defined as the sum of the squares of values in a matrix and are shown beneath each matrix in Fig. 2. The energy of the original image is preserved in the four submatrices A_1, H_1, V_1, and D_1 in a HWT:

$$E_{A0} = E_{A1} + E_{H1} + E_{V1} + E_{D1} \tag{6}$$



200	150	100	50	50	100	150	200
200	150	100	50	50	100	150	200
200	150	100	50	50	100	150	200
200	150	100	50	50	100	150	200
200	150	100	50	50	100	150	200
200	150	100	50	50	100	150	200
200	150	100	50	50	100	150	200
200	150	100	50	50	100	150	200

Fig. 1. An 8 × 8 intensity image with a strongly vertical fabric.