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Research Paper

The combined scaled boundary finite-discrete element method: Grain breakage modelling in cohesion-less granular media

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ABSTRACT

A computational technique combining the scaled boundary finite element method (SBFEM) and the discrete element method (DEM) is developed. Both methodologies work in tandem to model two mechanisms i.e. grain-to-grain interaction via DEM; and breakage of individual grains via SBFEM. Both play important roles in characterising the response of granular soils. The combination of the two methods results in some advantages in computational flexibility and implementation in modelling grain breakage in granular materials. Parametric studies demonstrate the method's ability to reproduce stress-strain curves in bi-axial tests of granular rock-fills; and qualitatively predicts characteristics of grain breakage observed in laboratory tests.

1. Introduction

The particle size distribution is an important factor in many engineering applications employing granular materials such as rock-fill dams, pavements and railroad foundations. Experimental studies on specimens of granular materials e.g. [1–6] have reported on the role of particle breakage and its effects on the particle size distribution of the granular assembly, which in turn, influences the mechanical properties e.g. yield strength, compressibility and permeability observed in the macroscopic scale. Therefore, interpretation of the role of particle breakage is crucial towards understanding the physics that underpins the theoretical foundation of granular materials.

Experimental studies of particle breakage in granular materials have been reported since the 1960's e.g. [1,7]. Notwithstanding the labour, cost and time involved, it is also difficult to interpret the evolution of particle breakage through examination of post-test fragments and debris due to the small length and time scales involved during particle breakage [8]. Computational methods are usually used to complement experimental studies in order to elucidate the meso-mechanics of particle breakage mechanism. In this respect, particle based methods such as the discrete element method (DEM) introduced by Cundall and Strack [9] has been used to study the particle breakage mechanism from a computational mechanics perspective. Many studies on particle breakage using DEM has been reported in the literature e.g. [10–19]. In these studies, the soil grains are modelled by an agglomerate of ideally shaped sub-particles e.g. spheres or disks. The bond between each sub-particle is defined, for example, by a contact mechanics-based elastic spring model, which triggers breakage when the maximum stress in the bond is exceeded. The aforementioned studies have made inroads towards the understanding some of the mechanisms of particle breakage e.g. the effects of particle breakage on the energy dissipation during crushing and the influence of particle breakage on the soil fabric.

Despite the well-documented research on the particle breakage in granular materials using the DEM, open questions remain with regards to the appropriateness of the DEM in relation to the observed macroscopic behaviour of granular materials. Robertson et al. [12] reported that the macroscopic plastic behaviour, the critical state and the stress-dilatancy relationships observed in granular sands cannot be appropriately modelled by the DEM due to its inherent assumption of rigid particle behaviour. The approach used by the DEM in modelling fracture within a particle constrains the failure path along the boundaries of sub-particles. Combined with the use of idealized shape particles e.g. disks and spheres, a large number of particles are usually required to adequately model the morphology and breakage path within the grains. This to some extent, restricts the size of problems that can be solved using the DEM. Modelling of damage and failure with the DEM also necessitates the use of particle-based bond/cohesive models e.g. [12] or by

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substituting individual rigid particles with a group of particles based on a pre-determined failure criterion [13]. These approaches usually require extensive calibration of the DEM models with experimental measurements [20].

The combined finite element-discrete element method (FDEM) is a hybrid computational method developed by [21] that exploits the advantages of the finite element method (FEM) e.g. [22,23] and the DEM. In this approach, all the particles are modelled within the framework of the DEM. Each discrete element is in turn, discretized into a finite element mesh. The elements in the mesh are connected at the inter-element boundaries by spring/cohesive elements following a pre-determined cohesive law. The contact interaction forces obtained from a DEM analysis are used to determine its deformation of a particle within the framework of the FEM. Unlike the DEM, the FDEM enables the deformation of each particle in the granular assembly to be determined explicitly via a full stress analysis. Fracture within a particle is constrained to take place along the inter-element boundaries when the deformation results in a stress state that exceeds the bond strength between adjacent finite elements. The success of the FDEM in modelling this transition from continuum to discontinuum through fracture and fragmentation is reported in applications in concrete [24], rock [25,26] and laminated glass [27]. Bagherzadeh-Khalkakhali et al. [28,29] further developed the FDEM to model particle breakage in rock-fills. In this approach, the breakage path of a particle is determined via a least squares fit from 'plastic' points satisfying a failure criterion and eliminates the constraint that crack propagation is allowed only along inter-element edges. The development of the FDEM has matured to a stage where parallel computing is achievable through virtual parallel machines that leads to very high efficiency for grand scale computations [30].

This manuscript describes the development of a computational technique that borrows the key ideas from the DEM of Munjiza [21] and combines the scaled boundary finite element method (SBFEM) of [31]. Since its introduction, the SBFEM has been applied to many fields in engineering. The method is perhaps most well-known in applications involving fracture e.g. [32] and wave propagation in unbounded domains e.g. [33]. In fracture mechanics, the semi-analytical nature of the SBFEM facilitates accurate modelling of the asymptotic stress field in the vicinity of any kind of stress concentrators [34]. This eliminates the need for local mesh refinement as would be required in the FEM. In problems involving unbounded domains, the SBFEM can accurately model the far-field radiation damping, which significantly reduces the size of the computational domain. The SBFEM has also been applied to other engineering applications including stochastic structural analysis [35], meso-scale modelling [36], fluid mechanics [37], heat transfer [38] and piezoelectric materials [39].

The redevelopment of the SBFEM with polygonal elements [40] exploits the flexibility of the arbitrary sided polygons leading to simple yet efficient remeshing algorithms for crack propagation modelling e.g. [41–43]. This development has also enabled the theory of the SBFEM to be extended to model heterogeneous materials [44] and nonlinear materials [45]. For general problems in elasticity, the SBFEM was shown to have improved accuracy and convergence behaviour compared to the other available polygonal elements in the literature [46]. For elastoplastic analysis, the SBFEM is computationally more efficient where higher accuracies are desired [45].

The technique resulting from combining the SBFEM and the DEM is known as the scaled boundary finite-discrete element method (SBFDEM) and is aimed at exploiting the advantages of both SBFEM and DEM with a particular focus on modelling particle breakage in granular materials. Within the framework of the SBFDEM, a particle is modelled as a polygon within the framework of both the DEM and the SBFEM. Only a single polygon is necessary for the stress analysis and a further discretization into sub-particles is not necessary. Particle interaction through contact is modelled within the framework of the DEM. A stress analysis is carried out in each polygonal particle using the SBFEM to determine its state of stress and check for possible breakage. The breakage path, as a first approximation, is assumed to be straight once the proposed breakage criterion is satisfied. When a particle breaks, two new particles are generated, replacing the original one.

Compared with the FDEM, the SBFDEM can potentially reduce the size of the problem that is to be analyzed. In the FDEM, a particle has to be modelled by a mesh of finite elements. This potentially increase the size of the problem through an increase in the number of degrees-of-freedom. During contact detection and contact force evaluation, the contacting couples of each particle in the FDEM are represented by a set of couples of finite elements. This further increases the total number of contacting couples to be considered, and in effect, the computational effort required. The SBFDEM requires that a particle be modelled by a polygon without further discretization. The SBFDEM also facilitates a simple procedure by which grain breakage can be modelled. The splitting of a polygon into two requires only minimal effort as the new polygons can be modelled directly by the SBFEM and DEM. If a FDEM approach is used, particle breakage has to be modelled using either a large number of finite elements or the use of complex algorithms to handle the ever changing topology and size of the problem.

The detailed computational aspects of the SBFDEM will be presented in the subsequent sections. This study is limited to two-dimensions. In Sections 2 and 3, the theories of the SBFEM and the DEM are briefly reviewed. The theoretical foundation and computer implementation aspects of the SBFDEM focusing on particle breakage modelling are described in Section 4. Section 5 presents parametric studies conducted with the SBFDEM to study the effects of particle breakage on the particle size distribution and the macroscopic mechanical behaviour of specimens of granular rock-fills under bi-axial loading conditions. The major conclusions are summarized in Section 6.

2. The scaled boundary finite element method

This section provides a brief summary of the SBFEM. Only the information necessary for the implementation of the SBFDEM are presented. The reader is referred to standard SBFEM texts published in the literature e.g. [31,47] for a complete documentation of the methodology.

The SBFEM is a semi-analytical technique developed by Song and Wolf [31]. The formulation is sufficiently flexible such that it can be formulated on polygons with arbitrary number of sides [40]. The geometry of a polygon is required to satisfy star convexity. All convex polygons automatically satisfy this condition. Where a polygon does not satisfy star convexity, subdivision is always possible to generate two or more corresponding convex polygons. Fig. 1 outlines the SBFEM coordinate of a generic polygon. A point (x_0, y_0) known as the scaling centre is defined at the geometric centre of the polygon. A radial coordinate system $0 \leqslant \xi \leqslant 1$ is defined with $\xi = 0$ at the scaling centre and $\xi = 1$ at the polygon boundary. The boundary of the polygon is discretised by one-dimensional finite elements with local coordinates (ξ, η).

The Cartesian coordinates $(x(\xi,\eta), y(\xi,\eta))$ of a point in a triangular sector covered by a line element on the polygon boundary are defined by

$$\begin{Bmatrix} x(\xi,\eta) \\ y(\xi,\eta) \end{Bmatrix} = \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} + \xi \mathbf{N}_u(\eta)\mathbf{x}_b \tag{1}$$

where the shape function matrix $\mathbf{N}_u(\eta)$ is defined as