Construction and Building Materials 146 (2017) 114–121

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

The application of general self-consistent model on mechanical behaviour of fibre-reinforced cementitious composites

H. Tian [a], Y.H. Cui [b], Y.X. Zhang [a,*], C. Yang [c]

[a] School of Engineering and Information Technology, The University of New South Wales, Canberra, ACT 2600, Australia
[b] School of Mechanical Engineering, Tianjin University, Tianjin, China
[c] School of Computing, Engineering and Mathematics, University of Western Sydney, Penrith, NSW 2751, Australia

HIGHLIGHTS

- Study of a new analytical model for FRCCs based on the general self-consistent method.
- Study of a hexagonal-shaped representative element model with multiple micro-cracks.
- Effective evaluation of the equivalent Young's modulus of a typical FRCC before and after cracking.
- Study of the effect of fibre orientation and the crack density of the matrix on Young's modulus of the FRCC.

ARTICLE INFO

Article history:
Received 14 November 2016
Received in revised form 26 March 2017
Accepted 7 April 2017

Keywords:
Analytical model
Equivalent Young's modulus
Fibre-reinforced cementitious composite (FRCC)
Mechanical properties
Microstructure

ABSTRACT

Fibre-reinforced cementitious composite (FRCC), by adding short discrete fibres randomly in cementitious composites, exhibits substantially improved mechanical properties than conventional cementitious composites due to the fibre bridging action and the existence of multiple micro-cracks. In this paper, based on the general self-consistent method a new micromechanical analytical model is developed to model the material properties of the FRCCs. A hexagonal-shaped representative volume element model with multiple micro-cracks is established based on the microstructure of the FRCC to represent the transversely isotropic material characteristics. The developed model is used to evaluate the equivalent Young's modulus of a typical FRCC before and after cracking, and the results obtained are compared to those obtained from other analytical models and experimental data for validation. The influence of the aligned orientation of fibre and the crack density of the matrix on Young's modulus of the composite is also studied.

1. Introduction

Cementitious composites are composed of Portland cement, fine aggregate, superplasticizers, and admixtures. Fibre-reinforced cementitious composite (FRCC) is a special type of cementitious composites with short fibres dispersed randomly in the cementitious matrix [1]. In particularly high performance FRCCs, such as engineered cementitious composite (ECC) exhibit substantially improved mechanical properties with a significantly improved tensile strain hardening capability than conventional cementitious composites [2–4]. The interactions between the fibre and cementitious matrix, the fibre bridging actions and the existence of multiple microcracks are the key for the superior mechanical behaviour for the high performance FRCCs [5–7].

Numerous researches have been conducted on FRCCs with most focusing on the experimental studies [8–12]. Experimental studies can exhibit the overall mechanical properties of the FRCCs, but the physical mechanism leading to the mechanical properties of the composites could not be obtained directly through experimental results. A few numerical studies have also been reported to predict the mechanical behaviour mainly the uniaxial tensile behaviour of the FRCCs, such as the continuum damage methanics model [13–15]; the finite element analysis by Spagnoli [16] and Lars [17], the stochastic finite element analysis by Kabele [18] and the extended finite element analysis by Huang et al. [19]. The crack behaviour through a crack bridging model which relates the bridging stress provided by fibres to the crack opening, have also been studied through various methods [20–23], and they are the most

---

[*] Corresponding author.
E-mail address: y.zhang@adfa.edu.au (Y.X. Zhang).

http://dx.doi.org/10.1016/j.conbuildmat.2017.04.045
0950-0618/© 2017 Elsevier Ltd. All rights reserved.

the dominant factors for the extraordinary tensile-hardening capability of the FRCCs in the numerical modelling.

To obtain a more insightful understanding of the physical mechanism for the superior mechanical behaviour of the FRCCs, analytical studies have also been conducted. Based on the conventional methods with most focusing on the prediction of the equivalent Young's modulus of the materials before and after cracking Najm and Naaman [3] predicted the Young's modulus of the FRCC with only the effect of air void and fibre volume fraction considered in the model. The analytical model for short fibre-reinforced polymers which considered the influence of fibre orientation was firstly proposed by Fu and Lauke [6] and Fu et al. [7] and then more analytical models on FRCC considering the influence of short fibre orientation were reported [17,18] [24–26]. In these studies, the FRCC is considered as a homogeneous material, however, for a FRCC which comprises of various components such as fibre, cementitious matrix, fine aggregate, the influence of the microstructure on the Young's modulus of the composite has not been considered, leading to an inaccurate prediction. To predict the mechanical properties of the FRCC accurately, the FRCC should be regarded as a multi-phase material and the effect of each phase such as fibres, matrix and microcracks should be considered in the model. However according to authors' investigation very few effective analytical models, which consider the effects of fibres, matrix and cracks have been reported for the multi-phased FRCCs.

To estimate the overall properties of FRCCs, various methods have been proposed, such as the simplified methods [27,28], the differential method [29], the composite sphere model, the strain energy approach (SEA) [30], the self-consistent method (SCM) [31,32], the general self-consistent method (GSCM) [33–35], and the Mori-Tanaka approach [36–40]. Among all these methods, the GSCM was found to be able to better describe the physical behaviour in the limiting case of a concentrated polydispersed suspension [29,30,41] and can better present the mechanical behaviour of FRCCs.

In this paper, a micromechanical analytical model is developed to model the mechanical properties of the FRCCs based on the GSCM model, which has been demonstrated to be effective to predict the mechanical properties of the FRCCs. A hexagonal-shaped representative volume element (RVE) model with multiple micro-cracks is established based on the microstructure of the FRCCs to represent the transversely isotropic material characteristics. The developed analytical model is applied to evaluate the Young's modulus of the FRCCs, and the results obtained are compared to those obtained from other analytical models and experimental data for validation. The effects of aligned angle of the fibre and crack density on the equivalent Young's modulus of composites are also investigated based on the new analytical model.

2. Equivalent moduli for FRCCs

The definition of equivalent moduli of the heterogeneous materials was discussed by Hashin and Rosen [30] and Roscoe [29]. Based on the GSCM, the fibre-reinforced cementitious composites can be considered as transversely isotropic materials when the phases of fibre and the surrounding matrix are hypothesised as isotropic materials. The equivalent moduli of the FRCCs are discussed herein.

The general Hooke's law for an anisotropic medium is given by

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \tag{1}$$

Based on the GSCM, the constitutive relation for an equivalent medium of the transversely isotropic materials [26] is expressed as

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{22}-C_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2C_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{23} \\ \varepsilon_{31} \end{bmatrix} \tag{2}$$

This elasticity stiffness matrix of the transversely isotropic materials has 5 equivalent medium constants, C_{11}, C_{12}, C_{22}, C_{23}, and C_{44} and they are the combinations of the elastic moduli of fibre and the surrounding matrix.

Five equivalent moduli, E_{11}, G_{12}, G_{13}, K_{23}, G_{23}, are used to identify the material properties of an transversely isotropic composite. According to the previous research [42], these moduli are combinations of the equivalent medium constantsC_{11}, C_{12}, C_{22}, C_{23}, and C_{44}, and given by

$$\begin{cases} E_{11} = E_1 = C_{11} - \frac{2C_{12}^2}{C_{22}+C_{23}} \\ G_{12} = G_{13} = G_1 = C_{44} \\ K_{23} = \frac{1}{2}(C_{22}+C_{23}) \\ G_{23} = \frac{1}{2}(C_{22}-C_{23}) \\ \gamma_{12} = \frac{1}{2}\left(\frac{C_{11}-E_{11}}{K_{23}}\right)^{1/2} \end{cases} \tag{3}$$

in which E_{11} is the Young's modulus along the longitudinal direction of the fibre, i.e. the direction of x_1. K_{23} and G_{23} are bulk and shear modulus of the composite in a direction that is perpendicular to the longitudinal direction of the fibre and they govern the plane-strain deformation in the x_2-x_3 plane. G_{12} is the shear modulus governing shear in any plane normal to the transverse x_2-x_3 plane; and γ_{12} is the longitudinal Poisson's ratio.

The equivalent moduli for equivalent materials are determined by identifying the equivalent medium constants through the equation of strain energy obtained by Eshelby [43]. The strain energy for a homogeneous medium containing an inclusion under the applied displacement condition is determined by [43]

$$E_{equiv} = E_0 - \frac{1}{2}\int_{\Gamma}(f_i^0 u_{ie} - f_{ie} u_i^0)\,ds \tag{4}$$

where E_{equiv} is strain energy for the equivalent homogenous medium, E_0 is the strain energy for the medium without an inclusion subjected to the same surface tractions, Γ is the surface of the inclusion, f_i^0 and u_i^0 are the tractions and displacements for the medium containing no inclusion, and f_{ie} and u_{ie} are the tractions and displacements at the same point in the medium containing inclusions.

3. A micromechanical RVE model for FRCCs

With the assumption of transversely isotropic material, the microstructure of the FRCCs can be represented by a 3-dimensional model, which is composed of two phases, i.e. fibre and cement paste as shown in Fig. 1.

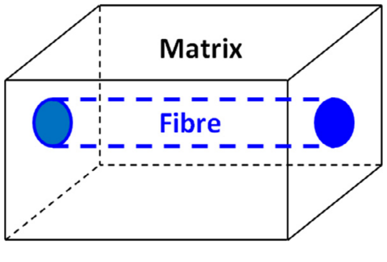

Fig. 1. A 3-D two-phase microstructure of a FRCC.