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The fractal evolution of particle fragmentation under different fracture energy

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ABSTRACT

Brittle particles fragment when compressed or under impact. The particle fragmentation leads to a power-law relation of the number or mass of particles as a function of the particle size. The resulting power-law relation is accurately characterised by a fractal dimension of the particle fragmentation. The fractal dimension of particle fragmentation depends on the probability of particle failure, which correlates with the applied stress and fracture energy according to Weibull's statistics. Thus, the fractal dimension of particle fragmentation is related to the applied stress and fracture energy, and the relation of the fragmentation fractal dimension to the applied stress and fracture energy is proposed in this paper. It can be validated by the experimental data of marble particle fragmentation and brittle fracture data for rock particles published previously.

1. Introduction

Particle fragmentation is abundant in nature and many fields of engineering, thus attracting a continuous interest in scientific and engineering research over the past decades [1,2]. Fragmentation phenomena can be observed on a broad range of length scales ranging from the collisional evolution of asteroids and meteor impacts on the astrophysical scale, through geological phenomena and industrial applications on the intermediate scale down to the break-up of large molecules and heavy nuclei on the atomic scale [3–5]. The most striking observation on the particle fragmentation is that the distribution of fragment sizes shows a power-law behavior, independently on the way of imparting energy, relevant microscopic interactions and length scales involved, with an exponent depending only on the dimensionality of system [6,7]. Preliminary work has shown that the dispersed progeny of the particle fragments could be accurately characterised by a scale-invariant distribution known as the fractal dimension of the particle fragmentation [8,9], a parameter simply drawn from the power-law exponent. Fragmentation fractal dimension is a ratio providing a statistical index of the fragmentation degree of particles [10]. Fragmentation fractal dimension is variable, and it increases with the intensity of the fragmentation process [11,12].

Several theoretical models have been proposed linking the fragment size distribution to the stress level and fracture energy [10,13–16]. Korvin [17] classified the fractal models of the particle fragmentation as either energy- or probability-based. The fractal dimension of the fragment size distribution is related to the energy required for comminution in the energy-based approach [18–20]. Nagahama [21] and Yong and Hanson [22] derived theoretical expressions for fragment size distribution as a function of energy density. Diehl et al. [23] revealed that a systematic dependence of the fragmentation fractal dimension on the input energy was evidenced at lower energies. In the probability-based approach, Perfect [3] revealed that the hierarchical failure in nature of particles, resulting that the number-size distribution of fragments depends on the probability of failure ($f(1/b^i)$ at each level in the hierarchy. Here, $f(1/b^i) = n_i/b^3$, and n_i is the number of cubic fragments produced from the aggregated subunits. In the case of scale-invariant $f(1/b^i) < 1$, the fragmentation fractal dimension correlated with the probability of failure is given as follow $D = 3[1 + \log f/\log n]$, for classical particles [3], where D is the fragmentation fractal dimension, and n is the number of fragments, and $n = 1/\alpha^3$, and α is a scaling factor. Extensive laboratory investigations have been conducted to correlate fragmentatiom fractal dimension with the stress level and fracture energy [5,21–27]. Investigations in the literature dedicated to study the evolution of fragmentation fractal dimension with the applied stress and fracture energy are very scarce. The objectives of this study were to: (i) clearly state the method to determine the fragmentation fractal dimension using the fragment size distribution, (ii) determine the relationship between the fragmentation fractal dimension and the applied stress or fracture energy. The fragmentation fractal dimension is calculated by the progeny fragments, whose diameters are less than the minimum diameter of the mother particles. The function formula of fragmentation fractal dimension is expressed by the applied stress or fracture energy. Drop weight tests on marble particles were implemented under different impact energies. Fragmentation fractal dimension of marble particle was calculated from the sieving data of the fragment size distribution under different impact energies. The relationship between fragmentation fractal dimension and the applied stress or

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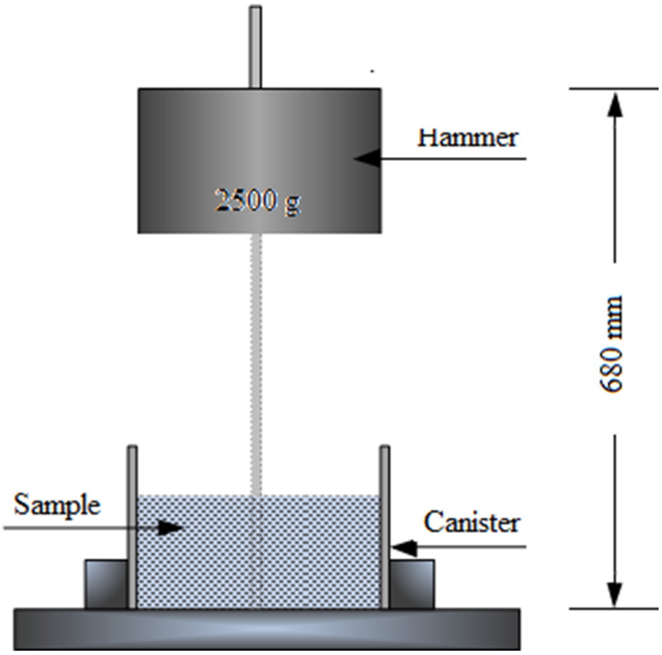

Fig. 1. Schematic view of the drop weight test equipment.

fracture energy is also validated using published data for rock particle fragmentation.

2. Theory

Turcotte [28] proposed a fractal model for the particle fragmentation based on the notion of a scale-invariant cascade of fragmentation. In this model, a parent material is composed of a population of structural units of fixed size d_0. A fraction f of this mass fragments into smaller units of size $d_1 = \alpha d_0$, where α is a constant <1, and usually set to 0.5 [29]. A fraction f of this mass in turn fragments into units of size $d_2 = \alpha d_1$. The division process is repeated across a range or cascade of scales, resulting in a geometric progression of fragment sizes. If M_i is the cumulative mass of fragments $\le d_i$, then the model may be written simply as $M_{i+1} = fM_i = f^i M_T$. A power-law relation yields in terms of fragment size d_i as follow

$$\frac{M(\le d_i)}{M_T} = \left(\frac{d_i}{d_0}\right)^{\delta} \tag{1}$$

where $\delta = 3 - D$, D is the fragmentation fractal dimension. The fragmentation fractal dimension is related to the probability of fracture in the form of

$$D = 3 - \frac{\log f}{\log \alpha} \tag{2}$$

where f is the probability of fracture. Peukert [30] derived the breakage probability of particles which are broken as a function of fracture energy in a fundamental way. The breakage probabilities of brittle particles are described with Weibull statistics. Weibull's brittle fracture equation is given by [31]

$$f = \left(\frac{\sigma}{\sigma_0}\right)^m \tag{3}$$

where σ_0 is the value of σ such that 37% of total number of test particles survival, f is the probability of failure (defined as the cumulative relative frequency distribution for σ), and m is a constant corresponding to the moments of the Weibull distribution, called the Weibull modulus. Substituting Eq. (3) into Eq. (2), the relationship between the fragmentation fractal dimension and the applied stress is given by

$$D = a + b \log \sigma \tag{4}$$

where $a = 3 + m \log \sigma_0 / \log \alpha$, $b = -m/\log \alpha$. The mass-specific energy is given by [32–34]

$$E_m = C\sigma^{\frac{5}{3}} \tag{5}$$

Combining Eq. (4) and Eq. (5) yields the relation of the fragmentation fractal dimension to the specific fracture energy as follow:

$$D = A + B \log E_m \tag{6}$$

where $A = 3 + m \log \sigma_0 / \log \alpha + 3m \log C/(5 \log \alpha)$, $B = -3m/(5 \log \alpha)$.

Fig. 2. Photograph of the drop weight test apparatus.