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Multiple-scale thermo-acoustic stability analysis of a coaxial jet combustor

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Abstract

In this paper, asymptotic multiple-scale methods are used to formulate a mathematically consistent set of thermo-acoustic equations in the low-Mach number limit for linear stability analysis. The resulting sets of nonlinear equations for hydrodynamics and acoustics are two-way coupled. The coupling strength depends on which multiple scales are used. The double-time-double-space (2T-2S), double-time-single-space (2T-1S) and single-time-double-space (1T-2S) limits are revisited, derived and linearized. It is shown that only the 1T-2S limit produces a two-way coupled linearized system. Therefore this limit is adopted and implemented in a finite-element solver. The methodology is applied to a coaxial jet combustor. By using an adjoint method and introducing the intrinsic sensitivity, (i) the interaction between the acoustic and hydrodynamic subsystems is calculated and (ii) the role of the global acceleration term, which is the coupling term from the acoustics to the hydrodynamics, is analyzed. For the confined coaxial jet diffusion flame studied here, (i) the growth rate of the thermo-acoustic oscillations is found to be more sensitive to small changes in the hydrodynamic field around the flame and (ii) increasing the global acceleration term is found to be stabilizing in agreement with the Rayleigh Criterion.

Keywords: Thermo-acoustics; Stability; Adjoint methods; Multiple-scale analysis; Sensitivity analysis

1. Introduction

Thermo-acoustic oscillations in gas turbines and rocket engines can lead to catastrophic failure and are one of the most persistent problems facing engine manufacturers [1]. They arise when acoustic pressure fluctuations occur sufficiently in phase with heat-release fluctuations. This converts thermal energy to mechanical energy over a cycle [2].

Thermo-acoustic oscillations are governed by the interaction between two macro subsystems: the combusting hydrodynamics and the acoustics. The hydrodynamics determines the

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flow field around the flame. The acoustics excites hydrodynamic structures at the flame base, which are then convected downstream, causing heat-release fluctuations. Although it has been argued that hydrodynamic instability has little influence on the thermo-acoustic stability, experiments [3] showed that the frequency of the coupled hydrodynamic/thermo-acoustic system is dictated by the hydrodynamic mechanism at some operating conditions and the thermo-acoustic mechanism at others. The interactions between the two subsystems is still not fully understood.

Linear stability analysis is often used to predict whether a combustion system will experience thermo-acoustic oscillations. The hydrodynamic subsystem is typically modeled by a response function [e.g. 4,5], or a time-delayed model [e.g. 1], relating the heat release to the acoustic velocity. These models are coupled with low-order acoustic solvers, such as network models or Helmholtz solvers [6], and the eigenvalues are calculated. If at least one eigenvalue has positive growth rate, the system is unstable. The sensitivity of the stability to external feedback mechanisms or passive control has been calculated with adjoint-based methods [7–9], which were recently experimentally validated [10]. However, these versatile methods were applied only to simplified thermo-acoustic models with uniform mean flows.

In most thermo-acoustic models, the interaction from the acoustics to the hydrodynamics is globally simulated by a simplified convection model, such as the $n-\tau$ model, meaning that the hydrodynamic variables are not included in the state vector. The system is only one-way coupled. To overcome this limitation, multiple-scale methods were applied by [11–13] to capture the two-way interaction between the hydrodynamics and acoustics. [11] used one time and two spatial scales (1T-2S) to couple a one-dimensional acoustic network model with a one-dimensional low-Mach number flame solver. Using the same multiple scales, [12] studied the nonlinearities in an electrical Rijke-tube with one-dimensional acoustics. The effect that the acoustics has on the hydrodynamics was assumed to be uniform in the hydrodynamic domain. [13] used two time and two spatial scales (2T-2S) in nonlinear simulations of a backward-facing step diffusion flame combustor to investigate the role of the acoustic Reynolds stress.

The main contribution of this paper is to develop and apply a three-dimensional thermo-acoustic linear model able to capture the two-way coupling between the hydrodynamics and acoustics. We apply the method to an axisymmetric coaxial jet combustor [14–16] with a diffusion flame. The chemistry is modeled by a one-step reaction but could be extended to detailed chemistry models. The *intrinsic sensitivity* is calculated with an adjoint method. This quantifies the growth rate's dependence on every interaction between the two subsystems. This directly gives the drift in the dominant eigenvalue caused by the global acceleration term, which couples the acoustics to the hydrodynamics, avoiding time-consuming nonlinear simulations [12]. The ultimate aim is to set up a robust framework for stability, receptivity and sensitivity analysis of combustion-acoustic systems and thereby to control thermo-acoustic instabilities by exploiting the behavior of hydrodynamic instabilities.

2. Governing equations

The non-dimensional compressible continuity, momentum, and energy equations are

$$\frac{\partial \rho}{\partial \tau} + \left(\frac{L}{h}M\right)\nabla\cdot(\rho\mathbf{u}) = 0, \tag{1}$$

$$\rho\frac{\partial \mathbf{u}}{\partial \tau} + \left(\frac{L}{h}M\right)\left[\rho\mathbf{u}\cdot\nabla\mathbf{u} + \frac{1}{\gamma M^2}\nabla p - \frac{1}{S_1 Re}\nabla\cdot\bar{\boldsymbol{\tau}}\right] = 0, \tag{2}$$

$$\rho\frac{\partial T}{\partial \tau} + \left(\frac{L}{h}M\right)\left[\rho\mathbf{u}\cdot\nabla T - \frac{1}{S_1 Re Pr}\Delta T - \rho Da Q_R + \frac{\gamma-1}{\gamma}\mathbf{u}\cdot\nabla p\right] + \frac{\gamma-1}{\gamma}\frac{\partial p}{\partial \tau} = 0, \tag{3}$$

where ρ is the density scaled by the ambient (oxidizer) density, $\tilde{\rho}_0$; $\mathbf{u}$ is the velocity scaled by the inlet fuel velocity, $\tilde{u}_f$; p is the pressure scaled by the ambient pressure, $\tilde{p}_0$; T is the temperature non-dimensionalized as $(\tilde{T}-\tilde{T}_0)/(\tilde{T}_f-\tilde{T}_0)$, where $\tilde{T}_f$ is the adiabatic flame temperature and $\tilde{T}_0$ is the ambient oxidizer temperature; Q_R is the rate of heat released by reaction; M is the Mach number; γ is the heat capacity ratio; Re is the Reynolds number based on fuel parameters, $\tilde{\rho}_f\tilde{u}_f h/\tilde{\mu}$, with $\tilde{\mu}$ being the dynamic viscosity; S_1 is the oxidizer-to-fuel density ratio, $\tilde{\rho}_0/\tilde{\rho}_f$; Pr is the Prandtl number $\tilde{\mu}\tilde{c}_p/\tilde{\lambda}$, where $\tilde{c}_p$ is the heat capacity at constant pressure and $\tilde{\lambda}$ is the thermal conductivity; Da is the Damköhler number $(\gamma-1)\tilde{\omega}_0 h/(\gamma\tilde{u}_0)$; $\tilde{\omega}_0$ is the reference reaction rate. We assume the Newtonian constitutive relation for the viscous stress tensor, $\bar{\boldsymbol{\tau}} = \nabla\mathbf{u} + (\nabla\mathbf{u})^T - 2/3(\nabla\cdot\mathbf{u})\mathbf{I}$, where $\mathbf{I}$ is the identity tensor. The reference length is the mean flame length, h, upstream of which it is assumed that most of the heat is released by chemical reaction. The flame acts on the acoustics as a compact heat source of finite spatial extent. The reference time is $L/\tilde{c}_0$, where $\tilde{c}_0$ is the reference speed of sound and L is the length of the combustor (Fig. 1). τ is the dimensionless acoustic time. We have neglected body forces and viscous dissipation effects in the energy equation, as a result of the low-Mach number limit. We have assumed Fourier's law for conduction, constant thermo-viscous properties, and a

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