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Effect of bending-twisting coupling on the compression and shear buckling strength of infinitely long plates

Christopher Bronn York[a,*], Sergio Frascino Muller de Almeida[b]

[a] Aerospace Sciences, School of Engineering, University of Glasgow, University Avenue, G12 8QQ Glasgow, Scotland, United Kingdom

[b] Department of Mechatronics and Mechanical Systems Engineering, University of São Paulo, Av. Professor Mello Moraes, 2231, São Paulo, SP 05508-030, Brazil

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ABSTRACT

This article describes the development of closed form polynomial equations for compression and shear buckling to assess the effect of *Bending-Twisting* coupling on infinitely long laminated plates with simply supported edges. The equations are used to generate contour maps, representing non-dimensional buckling factors, which are superimposed on the lamination parameter design spaces for laminates with standard ply orientations. The contour maps are applicable to two recently developed databases containing symmetric and non-symmetric laminates with either *Bending-Twisting* or *Extension-Shearing Bending-Twisting* coupling. The contour maps provide new insights into buckling performance improvements that are non-intuitive and facilitate comparison between hypothetical and practical designs. The databases are illustrated through point clouds of lamination parameter coordinates, which demonstrate the effect of applying common design heuristics, including ply angle, ply percentage and ply contiguity constraints.

1. Introduction

The effect of *Bending-Twisting* coupling continues to be ignored in studies relating to the buckling performance of plate or panel structures, which is broadly justified on the basis that the effects dissipate for laminates with a large number of plies. However, there is a significant body of research demonstrating that compression buckling strength may be overestimated (unsafe) and shear buckling strength may be overestimated or underestimated (over-designed) if the effects of *Bending-Twisting* coupling are ignored.

In this study, the effect of *Bending-Twisting* coupling on infinitely long laminated plates with simply supported edges is investigated, which complements an extensive literature on the subject, where the focus is primarily on finite length plates.

The relative buckling performance of adopting non-symmetric laminate designs is also revealed. With very few exceptions, the study of *Bending-Twisting* coupling effects has focussed entirely on symmetric designs.

Recent research has led to laminate design databases containing *Extension-Shearing* [1] and/or *Bending-Twisting* coupling [2]. The results have demonstrated that the design spaces contain predominantly non-symmetric stacking sequences. All are immune to thermal warping distortions by virtue of the fact that their coupling stiffness properties are null ($\mathbf{B} = \mathbf{0}$), as would be expected from symmetric laminate configurations. Heuristic design rules [3] are now applied to these databases to assess the effect on optimum buckling performance of practical rather than hypothetical designs. The reduction in the design space is readily quantified through graphical representation of the lamination parameter design space.

New insights into compression and shear buckling strength are provided via buckling factor contour maps, which are superimposed onto the lamination parameter design spaces. Contour mapping is applied to cross-sections through the design space, to allow detailed interrogation of the effects of *Bending-Twisting* coupling on buckling strength. The mapping is also applied to external surfaces of the feasible domain of lamination parameters, since these surfaces represent the bounds on buckling strength. The results are applicable to infinitely long plates with simply supported edges, which represent useful lower-bound solutions for preliminary design optimisation.

Notable contributions addressing infinitely long plates [4,5] adopted non-dimensional parameters, which differ from the lamination parameters used here. More importantly however, the buckling factor results presented were normalised by a bending stiffness parameter, which varies across the designs space, hence buckling performance is not directly comparable. Early studies on finite length plates have also adopted these non-dimensional parameters [6], as have the most recent studies [7], but a separate body of work has adopted lamination parameters [8–10] to aid optimum design. Comparisons with the

* Corresponding author.
E-mail address: Christopher.York@Glasgow.ac.uk (C.B. York).

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Nomenclature

$\mathbf{A}$, A_{ij}	extensional stiffness matrix and its elements, $i,j = x, y, s$
$\mathbf{D}$, D_{ij}	bending stiffness matrix and its elements, $i,j = x, y, s$
$E_{1,2}$, G_{12}	in-plane Young's moduli and shear modulus
H	laminate thickness (=number of plies, $n \times$ ply thickness, t)
N_x, N_s	compression and shear buckling load (N/mm)
k_x, k_s	non-dimensional buckling load factor for compression and shear
n	number of plies in laminate stacking sequence
Q_{ij}	reduced stiffness elements
t	ply thickness
U_E, U_G	laminate invariants for equivalent isotropic properties, $E_{\rm Iso}$ and $G_{\rm Iso}$
U_Δ, U_R	laminate invariants for orthotropic properties
x,y,z	principal axes
λ	buckling half-wave
ν_{ij}	Poisson ratio
ξ_Δ^A, ξ_R^A	lamination parameters for orthotropic extensional stiffness
$\xi_{\Delta c}^A$, ξ_{Rc}^A	lamination parameters for coupled extensional stiffness
ξ_Δ^D, ξ_R^D	lamination parameters for orthotropic bending stiffness
$\xi_{\Delta c}^D$, ξ_{Rc}^D	lamination parameters for coupled bending stiffness

infinitely long plate results of this study are therefore possible only for aspect ratios that correspond to the asymptotic result.

2. Design space interrogation

Ply angle dependent lamination parameters allow the stiffness terms to be expressed as linear variables within convenient bounds. However, the optimized lamination parameters must be matched to a corresponding laminate configuration within the feasible region, which is aided here by graphical representation of the lamination parameter design spaces [1,2]. In practical design however, heuristic rules are commonly applied, which generally involve constraints on ply percentages, ply contiguity and ply orientations [3].

Elements of the extensional stiffness matrix [**A**] are related to the lamination parameters by:

$$[\mathbf{A}] = H\begin{bmatrix} U_E + \xi_\Delta^A U_\Delta + \xi_R^A U_R & U_E - 2U_G - \xi_R^A U_R & \xi_{\Delta c}^A U_\Delta/2 + \xi_{Rc}^A U_R \\ U_E - 2U_G - \xi_R^A U_R & U_E - \xi_\Delta^A U_\Delta + \xi_R^A U_R & \xi_{\Delta c}^A U_\Delta/2 - \xi_{Rc}^A U_R \\ \xi_{\Delta c}^A U_\Delta/2 + \xi_{Rc}^A U_R & \xi_{\Delta c}^A U_\Delta/2 - \xi_{Rc}^A U_R & U_G - \xi_R^A U_R \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{xy} & A_{yy} & A_{ys} \\ A_{xy} & A_{ys} & A_{ss} \end{bmatrix} \quad (1)$$

where laminate invariants are defined in terms of the reduced stiffnesses:

$$\begin{aligned} U_E &= (3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})/8 \\ U_G &= (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})/8 \\ U_\Delta &= (Q_{11} - Q_{22})/2 \\ U_R &= (Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})/8 \end{aligned} \quad (2)$$

U_E and U_G are invariants associated with the equivalent isotropic properties of the laminate:

$$\begin{aligned} U_E &= E_{iso}/(1-\nu_{iso}^2) \\ U_G &= G_{iso} \end{aligned} \quad (3)$$

where E_{iso}, G_{iso} and ν_{iso} are the equivalent isotropic properties of the composite material, defined as:

$$\begin{aligned} E_{iso} &= 2(1 + \nu_{iso})G_{iso} \\ G_{iso} &= (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})/8 \\ \nu_{iso} &= (Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})/(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}) \\ &= 1 - 2U_G/U_E \end{aligned} \quad (4)$$

U_Δ is associated with orthotropy along axes 1 and 2, i.e. parallel and perpendicular to the fibre direction.

U_R has a complex physical meaning. It is a residual property contained in all elements of the stiffness matrix and is a function of A_{xx}, A_{xy} and A_{ss}. It also produces square symmetry, i.e., $A_{xx} = A_{yy}$, when the lamination parameter $\xi_\Delta^A = 0$. In this case, U_Δ is rendered zero, $A_{ss} = (A_{xx} - A_{xy})/2$ corresponds to $-\xi_R^A$ and, for off axis orientation of a laminate, β, containing standard ply angles $(0 + \beta)$, $(90 + \beta)$, $(45 + \beta)$ and $(-45 + \beta)$, $A_{xs} = -A_{ys}$. When $\xi_\Delta^A = \xi_R^A = 0$, $A_{ss} = G_{iso}H$.

The above equations are identical to the original equations [11]. Only the notation has been reformulated. The authors believe that this new notation is more intuitive, as it refers to the physical interpretation of the invariants and lamination parameters. Also, since there are only two material properties for an isotropic material, only two invariants (U_E and U_G) are used to describe the equivalent isotropic properties of the laminate; the original definition of the lamination parameters uses three invariants (U_1, U_4 and U_5) that are linearly dependent.

The ply orientation dependent lamination parameters are also related to the number of plies, n, by the following expressions:

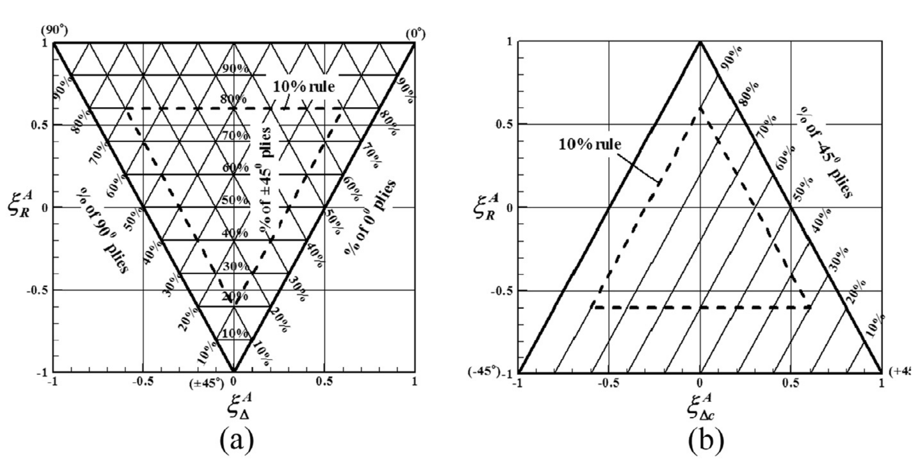

Fig. 1. Lamination parameter design space with ply percentage mapping for: (a) orthotropic stiffness (ξ_Δ^A,ξ_R^A) and; (b) anisotropic stiffness ($\xi_{\Delta c}^A$) relating to differing angle-ply percentages. The 10% design rule constraint is also illustrated.

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