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A variational model for free-edge interlaminar stress analysis in general symmetric and thin-ply composite laminates

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ABSTRACT

A variational model is developed to exactly determine both stress and displacement fields at free edges of general symmetric composite laminate strips under thermomechanical loads. By partitioning the total stresses in a composite with free edges into unperturbed (without free edge) and perturbation stresses and using the minimum complementary energy principle, the optimal stress and displacement fields are derived that exactly satisfy equilibrium, compatibility, boundary and continuity conditions. The paper extends the theory of stress-based variational stress transfer so that the effects of both applied traction and displacement loads can be considered. It has been also shown how the displacement field can be determined for a stress-based variational approach. The results are compared to refined finite element results. The superiority of the developed method over finite element method, both in terms of accuracy and computational efficiency, is discussed. The method is also applicable to thin-ply laminates and is computationally efficient.

1. Introduction

Stress concentrations like the free-edge effect may result in hazardous consequences since out of-plane or interlaminar stresses usually face quite weak material strength properties, which can eventually lead to premature failure due to corresponding interlaminar failure modes such as delamination. Although the free-edge problem is known from the 1970s, no analytical solutions exist that simultaneously satisfy the 3D elasticity governing equations along with all the traction free boundary conditions and the interface continuity conditions, due to the inherent complexities involved in the problem. Therefore, most approaches have been developed based on quasi-3D models or reduced 2D plate models [1] applied to a long rectangular composite laminate in which zero gradients along the axial coordinate are assumed. A comprehensive literature review on the methods proposed for determining the free-edge stress fields has been presented in Refs. [1,2]. Generally, approaches to the problem include the approximate elasticity solutions [3,4], different modifications of Pagano's theory [5–7] based on the Reissner variational principle, displacement-based equivalent single-layer (ESL) models [8], displacement-based layer-wise (LW) models [9], LW stress-based variational models [10,11] based on the minimum complementary energy principle, semi-analytical [12] and numerical [13] displacement-based approaches (e.g. FEM).

Among the developed analytical and numerical approaches, the LW stress-based variational method [10,11] based on the minimum complementary energy principle [14] is potentially more accurate for free-edge problems where there are zero traction boundary conditions. Contrary to displacement-based approaches, the approximate stress field derived with variational approaches satisfy exactly all the necessary equilibrium equations, interlaminar continuity and boundary conditions including zero-stress conditions at the free edges as well as the top and bottom surfaces of laminates. This approach was first introduced by Hashin [14] in 1985 for the analysis of cracked two-layer cross-ply laminates under uniaxial tension and further developed by many authors for analyzing cracked laminates with general lay-ups, see for example [15–18] as the most complete variational models developed so far. Kassapoglou and Lagace [10,19] were the first who applied the variational approach for analyzing stress concentrations near free edges. The reader is referred to the recent publications [1,2,16,17] for a detailed review of the developments in variational approach for the free-edge [1,2] and matrix cracking [16,17] stress transfer problems.

The main drawback of the available stress-based variational models is that they predict only stress components. It is due to this fact that the stress-based variational method based on the minimum complementary energy principle does not require the corresponding displacement field, nor does it relate the assumed stress distribution to the strain–displacement equations. Secondly, all of the developed variational models like other approaches discussed in this paper, have been formulated, not to completely resolve the free-edge or matrix cracking stress transfer problem, but just partially to eliminate the dependence of the

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stress field on the through-thickness coordinate of the laminate. Therefore, the interlaminar stress transfer problem can be reduced to a simple boundary value problem which is dependent only on the axial or transverse coordinate of the assumed laminate. For the variational free-edge stress transfer analysis [10,19] unlike the developed variational ply cracking models [15–18], this boundary value problem leads to a set of *non-homogeneous* differential equations as the Euler's equation(s) from the variational calculus. Indeed, the more layers considered, the more non-homogeneous equations will appear. Therefore, one difficulty in available variational models for the free-edge stress transfer problem is because of the increased number of non-homogeneous differential equations involved, which must be analytically solved. Moreover, there is not a systematic way to obtain and solve these governing equations and thus unlike the procedure introduced in Ref. [7], it is not possible to develop a software based on the current variational formulations. It simply means that the results obtained from the available variational models regarding the free-edge stress transfer [1,2,10,19] are approximate and thus, do not reproduce very refined finite element results. In addition, due to implementation of the minimum complementary energy principle, all of the developed stress-based variational models either for analyzing cracked laminates or free-edge stress fields are formulated for traction loading conditions. However, most of the experiments for free-edge analysis are performed under displacement loading conditions.

In this paper, a stress-based variational model is developed to accurately predict the complete state of stress and displacement in a general symmetric (possibly made of thin-ply) composite laminate containing free edges under uniaxial end stretching/shortening while the effects of thermally induced residual stresses are exactly taken into account. First, an admissible stress field containing some unknown functions is constructed that satisfies the stress equilibrium and all the traction boundary and continuity conditions. Then, by invoking the minimum complementary energy principle, the optimal stress state is achieved. An analytical approach is implemented to obtain the governing equations in a systematic way and consequently, the model can use the benefits of the ply refinement technique [7]. By partitioning the total stresses in a composite with free edge into unperturbed (without free edge) and perturbation stresses, it is possible to obtain a set of homogeneous fourth order differential equations with constant coefficients which are solved analytically. Moreover, it is shown how the strain and displacement fields can be determined when using a variational model based on the minimum complementary energy principle. The results obtained from the developed approach are compared to the available refined finite element results [7,20,21] for cross-ply, angle ply and quasi-isotropic laminates. The comparisons of stress and displacement fields under both mechanical and thermal loads show an excellent agreement. It should be mentioned that the assumed stress and displacement fields satisfy stress equilibrium, strain-displacement relations and all the boundary and continuity conditions and additionally, the minimum complementary energy principle and a ply refinement methodology are implemented to approach the exact solution. Moreover, the run time of the developed model is less than one second for all considered cases when using a double core Central Processing Unit (CPU) with 4 Gigabytes Random Access Memory (RAM) in a 64-bit operating system. Thus, for stress transfer analysis in symmetric laminates with free edges, the present approach has both high accuracy and computational efficiency.

2. Theoretical formulation

An arbitrary symmetric laminate having 2N perfectly bonded layers is considered. Since the laminate is symmetric, only the upper set of N layers will be considered, as shown in Fig. 1. A rectangular Cartesian coordinates system with the origin located in the mid length of the laminate, is selected. The x-coordinate defines the axial or loading direction, the y-coordinate defines the in-plane transverse (normal to free edges) direction and the z-coordinate defines the through-thickness direction. There are N-1 interfaces at the top half of the symmetric laminate ($z > 0$) which are shown by $z = z_i$; $i = 1, 2\ldots N-1$. The plane of symmetry (mid-plane) is shown by $z = z_0 = 0$ and the external top surface is specified by $z = z_N = h$. The total thickness of the laminate is 2h. The thickness of the ith ply is also specified by $h_i = z_i - z_{i-1}$. The orientation of the *i*th ply is demonstrated by the angle θ_i (measured counter clockwise) between the loading and the fiber direction of the layer. The stress and strain components and also material properties associated with the ith layer are denoted by a superscript or subscript i. It is assumed that the laminate is long (see Fig. 1) and can be infinitely extended in x directions ($L \gg w$) where 2L and 2W are, respectively, the length and width of the laminate.

The laminate is subject to external uniform strain ε_{xx} and a temperature difference ΔT. In a long and wide laminate without any free edges, the only nonzero stress terms are $\sigma_{xx}^{0(i)}, \sigma_{yy}^{0(i)}, \sigma_{xy}^{0(i)}$, where the superscript 0 defines the infinite or unperturbed laminate (without free edge) and the superscript (i), $i = 1,2,\ldots, N$, specifies the number of the ply. In this paper, the terms "infinite laminate" or "unperturbed laminate" refer to a long and wide laminate which does not have any free edges. Following the methodology used by Hashin [14] for analysis of cracked laminates, the stresses in the final geometry (with free edges) can be written as a superposition of the stresses in the unperturbed state and some perturbation stress functions (yet unknown) appeared due to the presence of the free edges.

$$\sigma_{mn}^{i}(X) = \sigma_{mn}^{0(i)} + \sigma_{mn}^{p(i)}(X). \tag{1}$$

where m,n = x,y,z and X = y,z. The first term in the right hand of Eq. (1) is the stress in the *i*th ply of the infinite laminate, which can be determined using an analysis based on the classical laminated plat theory. The second term in right hand of Eq. (1) is the perturbation stress in the *i*th ply, which unlike the stresses in the infinite laminate, is a function of location (X).

We aim to find admissible stress and displacement fields that satisfy the stress equilibrium equations $\sigma_{mn,n}^{i}(X) = 0$ and the following stress–strain-temperature relations:

$$\varepsilon_{xx}^{i} \equiv \frac{\partial u_i}{\partial x} = S_{11}^{i}\sigma_{xx}^{i} + S_{12}^{i}\sigma_{yy}^{i} + S_{13}^{i}\sigma_{zz}^{i} + S_{16}^{i}\sigma_{xy}^{i} + \alpha_1^{i}\Delta T, \tag{2}$$

$$\varepsilon_{yy}^{i} \equiv \frac{\partial v_i}{\partial y} = S_{12}^{i}\sigma_{xx}^{i} + S_{22}^{i}\sigma_{yy}^{i} + S_{23}^{i}\sigma_{zz}^{i} + S_{26}^{i}\sigma_{xy}^{i} + \alpha_2^{i}\Delta T, \tag{3}$$

$$\varepsilon_{zz}^{i} \equiv \frac{\partial w_i}{\partial x} = S_{13}^{i}\sigma_{xx}^{i} + S_{23}^{i}\sigma_{yy}^{i} + S_{33}^{i}\sigma_{zz}^{i} + S_{36}^{i}\sigma_{xy}^{i} + \alpha_3^{i}\Delta T, \tag{4}$$

$$2\varepsilon_{yz}^{i} \equiv \frac{\partial v_i}{\partial z} + \frac{\partial w_i}{\partial y} = S_{44}^{i}\sigma_{yz}^{i} + S_{45}^{i}\sigma_{xz}^{i}, \tag{5}$$

$$2\varepsilon_{xz}^{i} \equiv \frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x} = S_{45}^{i}\sigma_{yz}^{i} + S_{55}^{i}\sigma_{xz}^{i}, \tag{6}$$

$$2\varepsilon_{xy}^{i} \equiv \frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x} = S_{16}^{i}\sigma_{xx}^{i} + S_{26}^{i}\sigma_{yy}^{i} + S_{36}^{i}\sigma_{zz}^{i} + S_{66}^{i}\sigma_{xy}^{i} + \alpha_6^{i}\Delta T \tag{7}$$

where u_i, v_i and w_i define components of the displacement field for the *i*th layer in the x, y and z directions, respectively, and ε_{xx}^{i}, ε_{yy}^{i} etc., represent the infinitesimal strain terms. Moreover, the constants S_{ij} and α_i, respectively, show the compliance and thermal expansion coefficients of the *i*th layer. In addition, the admissible stress and displacement fields must satisfy the following boundary and interface continuity conditions:

1. Traction free condition on the top external surfaces $z = h$: $\sigma_{xz}^{N} = \sigma_{yz}^{N} = \sigma_{zz}^{N} = 0$.
2. Zero out-of-plane shear stresses σ_{xz} and σ_{yz} at the mid-plane $z = 0$ (due to symmetry with respect to the mid-plane).
3. Stress and displacement continuity conditions at the location of interfaces between the plies ($z = z_i$, $i = 1,2,\ldots,N-1$):

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