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# Effective elastic constants of wire mesh material studied by theoretical and finite element methods

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## ABSTRACT

Wire mesh is a high strength/stiffness material with versatility and little defect. This paper proposed a theoretical model to calculate the anisotropic effective elastic constants of a wire mesh material, and finite element method (FEM) is also carried out to validate the proposed model. Considering the effect of wire waviness and the discontinuity between the warp and weft wires, the analytic expressions of effective elastic modulus, shear modulus and Poisson's ratio were obtained. The results show a good agreement between the theoretical and FEM, revealing that the theoretical method gives a reliable prediction. The in-plane effective elastic modulus is higher about one order of magnitude than the out-of-plane modulus. Conversely, the out-of-plane shear properties are superior to the in-plane properties. The effective modulus are significantly affected by wire radius $R$, opening length $L$ and the ratio $R/L$. With the increase of $R/L$, the effective modulus of variant directions increases with different modalities. The wire waviness leads to much more in-plane stiffness-knockdown of wire mesh with thicker wires. Meanwhile, the out-of-plane stiffness is found to be weakened by the tiny contact area between the warp and weft wires. Stiffness reduction factors were proposed to describe the in-plane and out-of-plane stiffness-knockdown.

## 1. Introduction

Wire mesh is a kind of wire woven cellular material which can be stacked layer by layer as a structure to provide load supporting, filtration [1], mechanical impact absorption [1,2], heat transfer [2], and electrical energy storage [3]. To improve the thermofluid flow behavior and supply the electrical contact between components of solid oxide fuel cell (SOFC), the Nickel wire mesh is always used between the interconnector plate and the cell of SOFC as an anode contact layer [4–6]. Besides, the Nickel material of the wire mesh can contribute to the reactions because of the catalytic activity [7]. Wires have obvious merits as component of cellular structure, because they give high strength without defects and can be inexpensively produced and be handled easily during fabrication [8].

Recently, abundant wire woven cellular structures have been put forward such as WBK (Wire-woven Bulk Kagome) [9], WBD (Wire-woven Bulk Diamond) [10], WBC (Wire-woven Bulk Cross) [11], Strucwire [12] and Textile core [13]. The properties of the wire woven cellular structure are different from that of the parent material. Their properties are related with the geometry configuration. As for cellular structure, the earliest ones are honeycomb and foams. Gibson [14] in 1990s have proposed a model to analyze the effective elastic constants of honeycombs including effective elastic modulus, shear modulus and Poisson's Ratio, which are a function of wall length and thickness. Then, Gibson have investigated the mechanics of hollow sphere foams [15], and compared the properties of different packing: FCC (face-centered cubic), BCC (body-centered cubic) and SC (simple cubic) packing [16]. They concluded that the effective elastic modulus and strengths of the former one are higher than that of the latter two. After that, lots of similar researches have performed on honeycomb [17–19], open cell foam [20,21], syntactic foam [22,23] and 2D cellular structure [24]. When it comes to wire woven cellular structures, most of the studies are about WBK and its derivative structure. Lee et al. [25] optimized the WBK by two separate ways based on the sandwich panel weight and the slenderness ratio of the WBK core, respectively. The properties of WBK-cored panel performed as well as honeycomb with a specified constraint. Besides, the optimized WBK core is superior to the optimized octet structure. They [26] also proved that the elastic modulus of compression and shear obtained by analytic solutions were accurate. The WBK-cored panel structure showed excellent ability in load

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capacity, energy absorption and deformation stability. Lee et al. [27] studied the shear properties of WBC including the strength and stiffness in two different directions using analytic solution, experiments and finite element simulations. They found that for a given weight the shear modulus and shear strength of WBC are superior to those of the WBK and WBD. Besides, the interior space of WBC can be used for other functions, for instance, storage or passage of fluids. Soon after, Lee et al. [8] introduced a semi-WBC and straight-WBC, in which the straight wires partly or totally replace the helically formed wires in the ordinary WBC to get preferable shear modulus and strength. The relative density, shear modulus and strength of the three WBCs were investigated by the proposed analytic solutions. They concluded that the waviness of the struts near the cross leads to strength degradation.

As for textile core, early in 2000s, Sypeck et al. [13] proposed a textile synthesis laminates, which is based on the single layer wire mesh, and then stack the mesh and join using a transient liquid phase to form the structure. They concluded that the stiffness and strength parallel to the wires exhibited a linear relationship with the relative density, and the textile synthesis laminates showed great application prospect for efficient heat exchange. Zok et al. [28] have analyzed the bending and shear behavior of textile-cores sandwich panel aiming to minimum the weight. They found that the textile-cores sandwich structure with a wave angle of 45° exerts optimal performance in bending, shear, compression and their combinations. After that, Zupan et al. [29] investigated a similarly structure consisting of wire meshes, but the orientation of the studied properties exists an angle between the direction of wires. They discovered that, the stiffness and strength of the structure is determined by the axial stretch of its struts, and has a linear relationship with the relative density. Douglas T. [30] have studied the effects of truss waviness on the stiffness and strength of the wire mesh synthesis structure. Analytical and finite element method are performed to predict the compressive behavior of collinear and textile wire mesh. Compared with the collinear structure, the wire waviness of square textile wire mesh significantly reduces its stiffness and strength.

To sum up, the previous works are all about the properties of wire mesh in the same plane as wires. For the wire mesh material used in SOFC, its geometrical configuration affects the stress distribution of the SOFC [7]. Due to the complexity of structure, there is no theoretical formula to calculate the stress, and finite element method (FEM) is widely used at present. But FEM will generate a large number of elements and nodes, resulting in a high cost of computation. In order to reduce the computation cost, the wire mesh is assumed to be an equivalent solid plate [31]. But the problem is that how to calculate the effective material properties is still unclear, which is the foundation of the thermomechanical modelling of SOFC [4]. In addition, for diesel soot combustion [32], the anisotropic performance of wire mesh is necessary for the evaluation of load capability. However, it is unclear how the wire mesh responds when it suffers from interactions in variant direction. Therefore, it is essential to study the anisotropy elastic properties of the wire mesh material. In this paper, the effective elastic modulus, effective shear modulus and Poisson's ratio in three main directions were studied by theoretical analysis, and the FEM was also performed to verify its accuracy. In addition the influence of the geometric size on effective elastic modulus and shear modulus was fully discussed.

## 2. Theoretical method

Because the wire mesh is a kind of discontinuous structure, the effective elastic properties are different in different directions and closely related with its configuration. During the analysis proceeding, the wire mesh was treated as a structure consisting of wavy truss segments and nodes. The responses of the truss segments under various load were studied to obtain the effective elastic constants of the wire mesh.

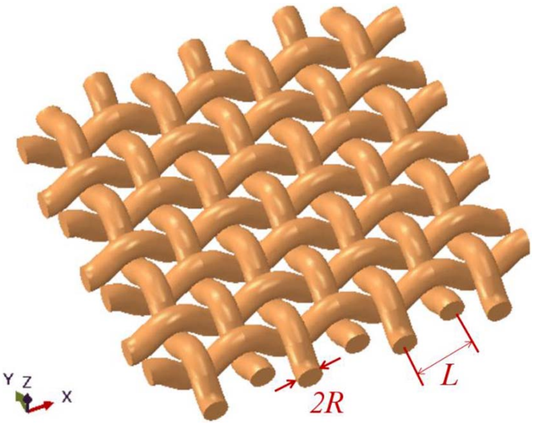

**Fig. 1.** Wavy wire mesh.

### 2.1. Relative density

Fig. 1 shows the schematic of a wire mesh material. The wavy wire mesh comprises the warp and weft wires, which should bend as they pass over each other. The radius of the woven cylindrical wire is $R$, and the opening width is $L$.

Because the wires are wavy in Z-direction, the profile of each wire can be described by

$$\tilde{z} = R\left(1 + \sin\frac{\pi x}{L}\right) \tag{1}$$

where $x$ is the axial coordinate along one wire measured from an intersection node of warp and weft wires. The length of wavy wire between two adjacent nodes can be calculated by

$$S = \int_0^{L/2} \sqrt{1 + (\tilde{z}')^2}\,\mathrm{d}x = \frac{\sqrt{L^2 + (\pi R)^2}}{\pi} \int_0^{\pi/2} \sqrt{1 - \frac{(\pi R)^2}{L^2 + (\pi R)^2}\sin\theta}\,\mathrm{d}\theta \tag{2}$$

Eq. (2) can be translated as

$$S = \frac{\sqrt{L^2 + (\pi R)^2}}{\pi} \int_0^{\frac{\pi}{2}} \sqrt{1 - K^2\sin^2\theta}\,\mathrm{d}\theta = \frac{\sqrt{L^2 + (\pi R)^2}}{\pi} E(K) \tag{3}$$

where

$$K = \sqrt{\frac{(\pi R)^2}{L^2 + (\pi R)^2}} \tag{4}$$

$E(K)$ is a kind of elliptic integral. For different value of $K$, the numerical solution of $E(K)$ can be obtained by consulting the elliptical integral table. Therefore, the relative density of the wire mesh material can be calculated by

$$\bar{\rho} = \frac{\pi R S}{L^2} \tag{5}$$

### 2.2. Out-of-plane properties

The out-of-plane effective elastic constants include effective elastic modulus $E_z'$, effective shear modulus $G_{xz}'$, $G_{yz}'$ and Poisson's ratio $\nu_{zx}'$, $\nu_{zy}'$. In the X-Y plane, the wire mesh is symmetric with respect to the X-axis and Y-axis. Therefore, there are relationships and $\nu_{zy}' = \nu_{zx}'$, and only three constants are needed to describe the out-of-plane properties of wire mesh material: $E_z'$, $G_{xz}'$ and $\nu_{zx}'$.

#### 2.2.1. Effective elastic modulus

Under load in Z-direction, the elastic response of the wire mesh material reflects the elastic modulus of basic material. Therefore, the effective elastic modulus of Z direction is determinated by the area of the load-bearing region. Considering the transverse plane of wire mesh