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Accurate simulation of delamination under mixed-mode loading using a cohesive model with a mode-dependent penalty stiffness

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ABSTRACT

The cohesive zone model approach in conjunction with a damage formulation, has been used by many authors to simulate delamination using finite element codes. However, some models available in the literature have not been validated correctly under mixed-mode loading conditions. An incorrect selection of the parameters of the model can result in inaccurate simulation predictions. In this work, the cohesive formulation previously developed by the authors has been updated with a mode-dependent penalty stiffness to ensure accurate and reliable simulation results. Different loading scenarios are simulated to validate the accuracy of the new formulation.

1. Introduction

Laminated fiber-reinforced composites may fail due to the development of intralaminar and/or interlaminar (delamination) damage mechanisms. Due to the relatively weak interlaminar properties of standard laminated composites, the anticipation of delamination onset and growth is needed in many application to ensure structural integrity. In practical applications, delamination grows under mixed-mode loading conditions. Therefore, accurate analysis models to predict delamination under mixed-mode loading are needed.

An effective method to analyse delamination is using cohesive zone models and so many different mixed-mode cohesive element formulations have been proposed [1–10]. Cohesive zone models provide an ideal representation of the delamination process of advanced composite materials. Two ingredients are needed for an excellent performance of a cohesive zone model. On the one hand, an accurate kinematics representation of the fracture process is needed. Methods based on the strong discontinuity of the displacement field are therefore used, either using cohesive elements [6,8] or formulations based on the extended finite element method [11–13]. On the other hand, a formulation of constitutive model that accurately describe the damage development under variable mixed-mode loading conditions. Despite this, two main difficulties concerning cohesive elements and their application at an industrial level still exist. Firstly, fine meshes are required to appropriately model the FPZ [14–16], which leads to high, and sometimes unaffordable, computational efforts. Secondly, most of the existing formulations, even those implemented in commercial finite element codes, have not been properly assessed under mixed-mode loading conditions and they can lead to inaccurate numerical results, since the models might not dissipate the energy correctly under variable-mode loading conditions [20]. When the fracture process zone is small enough for linear elastic fracture mechanics to be applicable, crack growth should be driven only by fracture toughnesses and the interlaminar strengths should not affect the dissipated energy. However, as observed by Sorensen et al. [17] and confirmed by Harper et al. [18], in some cases this does not hold true and the interlaminar strengths severally affect the numerical results. This phenomenon was analyzed recently by the authors for different formulations [19] and a solution to solve the problem was proposed by the authors in [20]. The authors demonstrate that the problem can be solved by guaranteeing a non-negative energy dissipation (*i.e.* preventing the healing of the material) when the local mixed-mode ratio changes. This yields to a new condition for the material properties of the cohesive model [20]:

$$\frac{K_{sh}}{K_3} = \frac{\mathscr{G}_{Ic}}{\mathscr{G}_{IIc}}\left(\frac{\tau_{sh}^0}{\tau_3^0}\right)^2 \tag{1}$$

that relates interlaminar strengths (τ_3^0,τ_{sh}^0), pure-mode fracture toughnesses ($\mathscr{G}_{Ic}$,$\mathscr{G}_{IIc}$) and penalty stiffnesses (K_3,K_{sh}). The subscripts $(.)_3$ and $(.)_{sh}$ make reference to mode I and to mode II and III, respectively. It should be noted here that the terms in the RHS of Eq. (1) are material properties that can be measured in the laboratory, while the terms in the LHS are numerical properties needed when implementing the model in a finite element software using cohesive elements. Thus, if it is assumed that the penalty stiffness of the model is mode independent, as in [8], Eq. (1) reduces the number of material properties of the model,

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setting for example the interface shear strength as a function of the other material properties. However, this is a kind of engineering solution and some physics of the problem are lost with this assumption. Therefore, a reformulation of the cohesive zone model by incorporating a mode-dependent penalty stiffness is needed. This is the aim of this paper, where the formulation developed by the authors [8] is updated by introducing a mode-dependent penalty stiffness.

2. Formulation of the cohesive zone model

The formulation presented in this section is an update of the previous model developed by the authors in [8]. The boundary value problem and the kinematic equations of the problem are not repeated here. For further details, the reader is addressed to [8].

The constitutive behaviour is defined using a cohesive damage zone model that relates the mixed-mode equivalent traction, μ, to the mixed-mode equivalent displacement jump, λ, at the interfaces between plies. Damage initiation is related to the interfacial strength of the material, μ^o. The energy dissipated per surface area during the damage process is bounded by the Fracture Toughness of the material, $\mathscr{G}_c$. The constitutive law used in this work assumes linear softening during damage evolution [8]. The cohesive law uses an initial linear elastic response before damage initiation, as shown in Fig. 1. The linear elastic part is defined using a penalty stiffness parameter, K_B, that ensures a stiff connection between the surfaces before crack propagation.

The Helmholtz free energy by unit surface of the interface under isothermal conditions is divided into two terms:

$$\Psi(\mathbf{\Delta},\mathscr{D}) = \Psi(\mathbf{\Delta},\mathscr{D})_{\text{coh}} + \Psi(\mathbf{\Delta},\mathscr{D})_{\text{con}} \tag{2}$$

where $\Psi(\mathbf{\Delta},\mathscr{D})_{\text{coh}}$ and $\Psi(\mathbf{\Delta},\mathscr{D})_{\text{con}}$ refer to the cohesive and contact energy contributions, respectively. The vector $\mathbf{\Delta} = \{\Delta_1,\Delta_2,\Delta_3\}^T$ contains the displacement jumps between the two homologous points of the respective adjacent surfaces, and $\mathscr{D}$ is the scalar damage variable. The definition of the energy terms should be selected such as it yields to a unilateral and a symmetric constitutive behaviour for propagation of mode I and shear modes, respectively. The corresponding expressions are:

$$\Psi(\mathbf{\Delta},\mathscr{D})_{\text{coh}} = \frac{1}{2}(1-\mathscr{D})[\Delta_i K_{ij}\Delta_j - K_{33}\langle -\Delta_3\rangle^2] \quad (i,j = 1\ldots3) \tag{3}$$

$$\Psi(\mathbf{\Delta},\mathscr{D})_{\text{con}} = \frac{1}{2}[K_{33}\langle -\Delta_3\rangle^2] \tag{4}$$

where $\langle\,\rangle$ are the Macaulay brackets defined as $<x> = \frac{1}{2}(x+|x|)$, and δ_{i3} is the Kroenecker delta. K_{ij} are the components of the stiffness matrix. Applying Coleman's method [3], the constitutive equation reads:

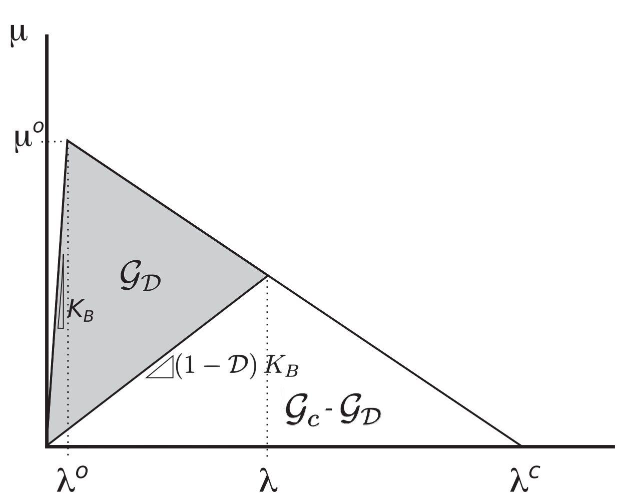

Fig. 1. Representation of the bilinear constitutive law used in the formulation for a fixed mode ratio.

$$\tau_i = \frac{\partial\Psi}{\partial\Delta_i} = D_{ij}\Delta_j \quad (i,j = 1\ldots3) \tag{5}$$

where D_{ij} is the constitutive tensor and reads:

$$D_{ij} = (1-\mathscr{D})K_{ij}\Delta_j + \mathscr{D}\delta_{i3}K_{33}\langle -\Delta_3\rangle \quad (i,j = 1\ldots3) \tag{6}$$

Note that the interface tractions can also be split between a cohesive and a contact term, $\tau_i = \tau_i^{\text{coh}} + \tau_i^{\text{con}}$, where:

$$\tau_i^{\text{coh}} = (1-\mathscr{D})[K_{ij}\Delta_j + \delta_{i3}K_{33}\langle -\Delta_3\rangle] \quad (i,j = 1\ldots3) \tag{7}$$

$$\tau_i^{\text{con}} = -\delta_{i3}K_{33}\langle -\Delta_3\rangle \quad (i,j = 1\ldots3) \tag{8}$$

The stiffness matrix, K_{ij}, is defined as a diagonal matrix. K_{33} is the penalty stiffness for mode I and K_{22} and K_{33} the penalty stiffnesses for shear modes. Note that the contact contribution is not affecting to the cohesive formulation. From now on, the interface traction τ_i denotes only to the cohesive component τ_i^{coh}.

2.1. Equivalent mixed-mode norms

To formulate the damage evolution law, mixed-mode norms of the tractions and the displacement jumps are defined. The mixed-mode equivalent traction μ is defined, like in the original mode [6,8], as the Euclidean norm of the individual tractions:

$$\mu = \sqrt{\tau_1^2 + \tau_2^2 + \langle\tau_3\rangle^2} \tag{9}$$

The mixed-mode equivalent displacement jump λ is imposing the following two equations:

$$\mu = (1-\mathscr{D})K_B\lambda \tag{10}$$

$$\Psi_{\text{coh}} = \frac{1}{2}\mu\lambda \tag{11}$$

where K_B is a mode-dependent penalty stiffness. The mixed-mode equivalent displacement jump and the mode-dependent penalty stiffness are obtained by substituting Eqs. (3) and (9) into Eqs. (10) and (11):

$$\lambda = \frac{K_{11}\Delta_1^2 + K_{22}\Delta_2^2 + K_{33}\langle\Delta_3\rangle^2}{\sqrt{K_{11}^2\Delta_1^2 + K_{22}^2\Delta_2^2 + K_{33}^2\langle\Delta_3\rangle^2}} \tag{12}$$

$$K_B = \frac{K_{11}^2\Delta_1^2 + K_{22}^2\Delta_2^2 + K_{33}^2\langle\Delta_3\rangle^2}{K_{11}\Delta_1^2 + K_{22}\Delta_2^2 + K_{33}\langle\Delta_3\rangle^2} \tag{13}$$

To completely define the evolution of the damage variable under mixed-mode loading, a local mixed-mode ratio B is defined as[6,8]:

$$B = \frac{\mathscr{G}_1 + \mathscr{G}_2}{\mathscr{G}} \tag{14}$$

where

$$\mathscr{G}_i = \int_0^{\Delta_i} \tau_i\, d\Delta_i \tag{15}$$

$$\mathscr{G} = \int_0^{\lambda} \mu\, d\lambda \tag{16}$$

Using Eqs. (13) and (12), the local mixed-mode ratio reads:

$$B = \frac{K_{11}\Delta_1^2 + K_{22}\Delta_2^2}{K_{11}\Delta_1^2 + K_{22}\Delta_2^2 + K_{33}\langle\Delta_3\rangle^2} \tag{17}$$

Finally, defining $K_{sh} = K_{11} = K_{22}$ and using Eq. (17) in Eq. (13), the mode-dependent penalty stiffness is condensed to:

$$K_B = K_{33}(1-B) + BK_{sh} \tag{18}$$

2.2. Damage activation function and evolution law

The damage activation function is defined as: