Computers and Geotechnics 82 (2017) 43–53

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Implicit integration of simple breakage constitutive model for crushable granular materials: A numerical test

Chunshun Zhang [a,b,*], Jian Ji [b], Sheng-Qi Yang [a], Jayantha Kodikara [b]

[a] State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou 221116, PR China
[b] Department of Civil Engineering, Monash University, Clayton, Vic 3800, Australia

ARTICLE INFO

Article history:
Received 21 June 2016
Received in revised form 15 September 2016
Accepted 29 September 2016

Keywords:
Breakage mechanics
Simple breakage model
Backward Euler return algorithm
FEA
Granular materials

ABSTRACT

In the context of the recently developed breakage mechanics that is based on thermodynamics principles, this paper presents a mathematical modelling procedure to implement the simple (i.e., linear elastoplastic) breakage constitutive model using finite element analysis (FEA) with illustrations by engineering applications. More informative mathematical derivation procedures of energy dissipations, plastic potential, yield function and non-associated flow rules are presented. In contrast, the existing relevant publications often lack sufficient elaboration, leaving knowledge gaps in the full understanding the model. This is followed by a series of numerical simulations in ABAQUS to test the model at the constitutive level. Various isotropic and triaxial shear tests in drained or undrained conditions are tested to illustrate the key features of the breakage model, which seem to be overlooked in the literature. Finally, a few numerical results are compared with experimental shear tests to demonstrate the ability of the simple breakage model in reflecting mechanical responses of crushable granular aggregates.

1. Introduction

Different stress return algorithms have been proposed to integrate the constitutive relationships to compute the increments in stresses and state variables. One of the relatively easy algorithms is the so-called 'explicit scheme'. This integration scheme enables the updated quantities at time $t + \Delta t$ to be calculated based on known quantities at time t. Its notable advantage is the simplicity in FE implementation and it has been widely used in geomechanics [1]. However, the yield condition is not guaranteed at time $t + \Delta t$ in such a forward integration process. As a result, the calculated quantities, for instance the plastic multiplier at time $t + \Delta t$, is not satisfied in the yield condition. This causes the solution over many increments to drift away from the yielding surface [2]. Moreover, the time step size, Δt, cannot be too large. Otherwise, incorrect results will be encountered. Therefore, the application of explicit schemes is usually limited to some simple constitutive models (e.g., linear elastic model). For complex non-linear constitutive models, the explicit scheme is usually neither efficient nor applicable despite that some researchers have been attempting to improve the performance of the explicit scheme. A good example is the explicit method with automatic substepping and error control as proposed by [3] for a suction-dependent unsaturated soil model, and more recently by [4] for integrating the well-known Barcelona Basic Model.

In contrast to the explicit scheme, another integration algorithm is called 'fully implicit scheme'. In detail, an (elastic) trial stress increment is firstly computed to obtain the updated trial stress $\sigma^{tr}_{t+\Delta t}$. The trial stress will be outside the yield surface if the yield condition is not satisfied. On this occasion, the trial stress is then brought back onto the yield surface at time $t + \Delta t$ with a plastic correction. This is known as stress return process that must be solved iteratively, of which a widely used method is the Newton method. Fully implicit method ensures the yield condition that is satisfied at each time increment, thus avoiding the deviation from the yield surface, as commonly encountered in the explicit scheme. In addition, the fully implicit scheme allows for the use of significantly larger time increments, which can give rise to faster solutions [2]. Note that the fully implicit method has been widely used in geomechanics [5–7] and general complex elastoplastic and viscoplastic materials, and models of which the mechanical responses are determined by all principal stress invariants.

The constitutive equations can also be integrated using a backward Euler return algorithm [6,8–14]. This might be considered to be a 'semi-implicit' method with first-order accuracy, as it still follows the idea of stress return but relies on the first-order of

* Corresponding author at: Department of Civil Engineering, Monash University, Clayton, Vic 3800, Australia.
E-mail address: ivan.zhang@monash.edu (C. Zhang).

http://dx.doi.org/10.1016/j.compgeo.2016.09.014
0266-352X/

Nomenclature

B	breakage index
D_{ijkl}	elastic matrix
D_m	minimum grain size
D_M	maximum grain size
e_{ij}	deviatoric strain tensor
E_B	breakage energy
$\overline{E_B}$	dissipative breakage energy
E_c	breakage energy constant
$F(x, B)$	current cumulative GSD by mass
$F_0(x)$	initial cumulative GSD by mass
$F_u(x)$	ultimate cumulative GSD by mass
G	shear modulus
K	bulk modulus
M	ratio of failure shear stress to the volumetric one
p	mean stress
$\overline{p}$	dissipative mean stress
p_0	initial mean stress
p_c	critical isotropic confining pressure
p_f	mean stress at failure
q	triaxial shear stress
$\overline{q}$	dissipative triaxial shear stress
q_f	triaxial shear stress at failure
s_{ij}	deviatoric stress tensor
T_{ijmn}	tangent stiffness matrix
x	grain size
y	yield function in mixed stress/breakage space
y^{trial}	yield function at elastic trial point
y^*	yield function in generalized dissipative triaxial stress/breakage space
α	fractal dimension
ε_v^e, ε_s^e	elastic volumetric and shear strains in triaxial conditions
ε_{ij}, ε_{ij}^e	strain and elastic strain tensors
ε_v, ε_s	volumetric and triaxial shear strains
δ	increment
δ_{ij}	Kronecker delta
δB	incremental breakage
$\delta\varepsilon_v^p$, $\delta\varepsilon_s^p$	incremental plastic volumetric and shear strains
$\delta\lambda$	non-negative plastic multiplier
ϑ	grading index
σ_{ij}	stress tensor
ω	plastic-breakage coupling angle
Ψ	macroscopic specific elastic strain energy
$\delta\Phi$	increment of dissipation potential
$\delta\Phi_B$	increment of breakage dissipation
$\delta\Phi_p^v$	increment of plastic volumetric dissipation
$\delta\Phi_p^s$	increment of plastic shear dissipation

Taylor's expansion of the yield function. It is much simpler than the fully implicit method regarding the stress return process because the iteration is not strictly needed to calculate the updated stresses. Therefore the semi-implicit method makes the FE implementation much easier compared with the fully implicit method. Moreover, the semi-implicit method enables quantities to be updated almost as accurately as those obtained from the fully implicit method. Due to its simplicity, stability and high level of accuracy, the semi-implicit integration method is used in this paper to implement the simple breakage model.

The objective of this paper is to numerically verify the implementation of the simple breakage model. To achieve this, fundamental features of the breakage model and its thermodynamically consistent constitutive equations are briefly reviewed in Section 2. Especially, the relevant issues in model derivations of the energy dissipations, plastic potential, yield function and non-associated flow rules are elaborated to avoid incompleteness of understanding, as often encountered in the existing literatures. Section 3 illustrates the FE implementation by a standard return mapping method. This shows the model's versatility in various numerical applications. Section 4 tests the efficiency of the FE implementation at the material point level under various conventional loading conditions. This helps to understand the fundamental features of the model, which seem to be overlooked in the literature. Then in Section 5, laboratory testing of some weak to hard granular materials are numerically modelled to demonstrate some simple application, where the experimental data are in good agreement with the numerical counterparts. Summaries and discussions are presented in the last section.

2. Discussion of the breakage mechanics and simple breakage model

The significance of thermodynamics in the formulation and development of constitutive models has been widely acknowledged [15–18]. A rigorous constitutive model must satisfy both the first and second laws of thermodynamics in order to be consistent and physically meaningful. Otherwise, a model that does not obey the thermodynamics framework may not be used confidently in describing a material behaviour [16]. The importance of obeying thermodynamics framework in developing constitutive models has been recently stressed by Al-Rub and Darabi [19] and Darabi et al. [20], who established a general thermodynamically consistent framework for coupling various mechanisms such as temperature, viscoelasticity, viscoplasticity, viscodamage, and micro-damage healing for constitutive modelling of time- and rate-dependent materials.

On the other hand, the formulation of continuum breakage mechanics and simple breakage model [21,22] were developed on the basis of thermodynamics principles. Following the thermodynamics principles, the breakage mechanics emphasises the significance of linking micro to macro scales (i.e., avoiding as much as possible the arbitrary mathematical structures) in constitutive modelling by incorporating grain size distribution (GSD) and its evolution (through breakage, B) to capture the macroscopic soil behaviour. In this way, the underlying microscopic process connects with macroscopic behaviour, which many other continuum theories fail to capture.

There are various thermodynamically consistent breakage models that have been derived from the breakage mechanics, providing physical explanations for many of the phenomenological aspects of crushable soil behaviour. Thus, these models have far been used in different engineering applications, including geophysics [23,24], rock mechanics [25], foundation engineering [26], unsaturated soil mechanics [27], and a general breakage model accounting for finite deformation and porous compaction and dilation [28]. Among those breakage models, the focus of the present study is the simple breakage model that is the simplest and fundamental form among all other relatively more complicated breakage models. According to the simple breakage model, the macroscopic specific elastic strain energy stored in a granular aggregate including particles of various sizes is simplified using linear elasticity [29]:

$$\Psi = (1 - \vartheta B)\left(\frac{1}{2}K\varepsilon_v^{e^2} + \frac{3}{2}G\varepsilon_s^{e^2}\right) \quad (1)$$