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Exact vibration modes of multiple-stepped beams with arbitrary steps and supports using elemental impedance method

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ABSTRACT

A novel numerical method has been developed and applied to the prediction of vibration modes of general stepped beams with arbitrary steps and general elastic supports. Each beam section is modelled as an element with input impedance at one end and output impedance at the other. These impedances are then coupled when the beam sections are joined to form an overall stepped beam structure. General elastic supports are modelled and their effects on impedance are examined. The method is theoretically exact assuming each beam section can be modelled as Euler-Bernoulli beam and is computationally very efficient since it does not involve any matrix operations of large dimension. Vibration modes of a number of stepped beams of different configurations have been computed using the proposed method and are compared with existing results in literature. Experimental investigations have also been carried out to validate the practical usefulness of the method. The method is also ideally suited for forced vibration applications since it not only establishes vibration modes, but also frequency response functions. In addition, a general purpose software has since been developed based on the method which can be very useful for structural dynamics design of general stepped beam structures.

1. Introduction

Beams have been frequently employed as the main structural members for many engineering structures and hence, their vibration characteristics have been of interests to many researchers for decades. Among the early pioneers of researchers, Timoshenko [1] investigated transverse vibration characteristics of uniform beams in 1922. Duncan devised a powerful vibration analysis method based on the concept of mechanical admittance and applied it to vibration problems of continuous beams [2]. The mechanical admittance method proposed by Duncan was then further developed to become later the mechanical receptence method and firmly established as a useful practical method of vibration analysis of beam and beam-like structures by Bishop et al. [3,4]. Alternatively, a more generalized receptance formulation for vibration analysis of beams was presented by Milne [5] and was further improved by including shear and rotary inertia effects by Stone [6]. Alongside with these studies, vibration analysis using mechanical impedance was examined [7–9] and was found to be useful. Mechanical impedance is defined as the matrix inverse of receptance matrix. Liang et al. [8] used an impedance method to analyse vibration of a beam with active piezoelectric actuators. Gatscher and Kawiecki [9] compared some mechanical impedance methods and showed that mechanical impedance methods can be effectively used to reproduce the field vibration environment in a laboratory test. These early impedance methods were developed primarily for the prediction of structural natural frequencies or frequency response functions and they fell short when detailed modeshapes are required to be identified since the degrees of freedom included in the impedance model are usually too few to enable modeshapes to be determined with decent spatial resolution. The number of degrees of freedom included in mechanical impedance analysis is usually limited to ensure efficiency and to avoid numerical ill-conditioning since inverse of a large receptance matrix is involved. On the other hand, matrix displacement method [10] has been used to formulate structural stiffness matrices in finite element analysis. However, mechanical impedance formulation [9] can be considered as a general dynamic version of the matrix displacement method which is a special static case when frequency $\omega = 0$ at which the impedance matrix becomes just the stiffness matrix. But for any dynamic analysis with frequency $\omega \neq 0$, mechanical impedance matrix is more accurate than the static stiffness matrix plus the mass matrix which has to be derived through other appropriate formulations. Both impedance methods

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and matrix displacement method have been successfully applied to vibration analysis of beam-like structures.

Beams with step changes in cross-sections frequently appear in mechanical and civil engineering structures and have since attracted much research interests among vibration community. Jiang and Bert [11] investigated vibration characteristics of beams with single step under various boundary conditions. The frequency equations can be derived in this case from a 4th-order determinant equated to zero and from which, free vibration modes can be solved. Based on similar concept of working with determinants, Naguleswaran [12,13] succeeded in deriving vibration modes of more complex geometries of beams with up to three step changes. Dong et al. [14] analysed vibration properties of stepped beams treated as Timoshenko beams with shear deformation and rotary inertia effects being included. When vibration of beams with distributed concentrated masses is concerned, Wu and Hsu [15] developed a numerical procedure which can be effectively employed to compute natural frequencies and mode shapes. Recently, much research interests have been directed towards the characterisation of free and forced vibrations of general multiple stepped beams based on more recent complex and sophisticated mathematical concepts. A composite element method which marries finite element method with classical beam theory was developed by Lu et al. [16] and found to be effective. The conventional differential quadrature method [17] was modified and further developed to become differential quadrature element method to be used for vibration analysis of multiple stepped beams [18]. A more general numerical procedure originally proposed by Adomian [19] for solving general physics problems was adopted by Mao et al. [20] to compute vibration modes of stepped beams. Similarly, a discrete singular convolution technique was adopted and applied to beam vibration analysis by Duan et al. [21] and found to be accurate. In addition, high-order modes of stepped beams were evaluated through a numerical procedure proposed by Xu et al. [22] which avoids some of the existing numerical difficulties in computing the higher mode shapes.

For dynamic analysis of stepped beams under various non-standard elastic supports, a number of methods have been developed to date to predict their vibration modes. Considering general elastic supports at both ends, de Rosa [23] presented a numerical method to compute vibration frequencies and mode shapes. Intermediate elastic supports of a clamped-clamped beam were analysed by Maurizi et al. [24]. Dynamics and stability of beams sitting on elastic Winkler foundations were presented by de Rosa [25]. Damped vibration problems of beams with elastic supports were also solved and the eigenvalue behaviours were examined [26]. Vibration problems of non-uniform beams with elastically restrained ends were analysed by Auciello [27]. A differential transformation method was developed and applied to stepped beams with elastic end constraints by Suddoung et al. [28].

Much research has been carried out and many methods have been developed to date to analyse vibration modes of beams with multiple steps and/or under general elastic supports. However, general and theoretically exact method which can be applied to analyse beam structures with any arbitrary steps and elastic supports has yet to be developed. In the present paper, a novel numerical method has been developed and applied to the prediction of vibration modes of general stepped beams with arbitrary steps and general elastic supports. Each beam section is modelled as an element with input impedance at one end and output impedance at the other. These impedances are then coupled when the beam sections are joined to form an overall stepped beam structure. Unlike existing mechanical impedance method [9] whose primary concern is to determine accurately vibration frequencies, or frequency response functions in the case of forced vibration, and which often leads to poor prediction of vibration modeshapes due to the generally limited number of degrees of freedom included in the analysis to ensure efficiency and to avoid numerical ill-conditioning due to the inverse of large receptance matrix, the proposed new method not only predicts very accurately vibration natural frequencies, but also very well defined vibration modeshapes which are generally required in structural dynamics design. The novelty of the method lies in the fact that once the natural frequencies are determined, these frequencies are then returned to each beam segment for very efficient and accurate identification of an individual and subsequently, a global modeshape. These modeshapes can be continuously defined since the receptances of each individual beam segment is exact and continuous. General elastic supports are modelled and their effects on impedance are examined. The method is theoretically exact assuming each beam section can be modelled as Euler-Bernoulli beam and is computationally very efficient since it does not involve any matrix operations of large dimension. Vibration modes of a number of stepped beams of different configurations have been computed using the proposed method and are compared with existing results in literature. Experimental investigations have also been carried out to validate the practical usefulness of the method. The method is also ideally suited for forced vibration applications since it not only establishes vibration modes, but also frequency response functions. In addition, a general purpose software has been developed based on the method which can be very useful for structural dynamics design of general stepped beam structures.

2. Preliminaries

The general structure to be tackled in this paper is a general beam structure with arbitrary beam sections of different geometries and generally different elastic supports. Each section is however assumed to be uniform whose vibration characteristics are assumed to have been fully solved under free-free boundary conditions. The full derivation of receptances of a uniform beam under different boundary conditions were discussed [5] and only a brief discussion is made here for the sake of self-completeness only for the case of free-free boundary conditions since that is what is required by the present elemental impedance method to be developed in this paper.

For a uniform beam shown in Fig. 1, the governing equation for the vibration deflection v can be written as,

$$\frac{EI}{\rho A}\frac{\partial^4 v}{\partial y^4} + \frac{\partial^2 v}{\partial t^2} = 0 \quad (1)$$

where E, I, A, ρ are the Young's modulus, second moment of area, cross-sectional area and density of the beam, respectively. Assume the vibration response v due to the force $Fe^{i\omega t}$ applied at $x = l$ as,

$$v = X(x)e^{i\omega t} \quad (2)$$

Upon substituting (2) into (1), one has,

$$\frac{d^4X}{dx^4} - \frac{\omega^2 \rho A}{EI}X = 0 \quad (3)$$

The general form of the function $X(x)$ satisfying (3) is therefore,

$$X = A\cos\lambda x + B\sin\lambda x + C\cosh\lambda x + D\sinh\lambda x \quad (4)$$

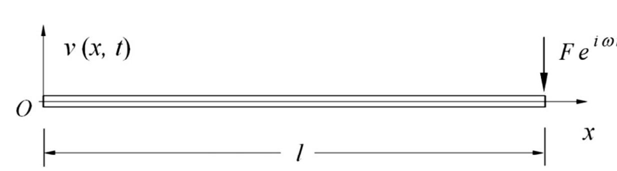

Fig. 1. A free-free uniform beam subject to an end force.