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Minimum thickness of semi-circular skewed masonry arches

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The problem of determining the minimum thickness of masonry arches has been a challenge to the engineering community through the last two centuries. Although significant work has been undertaken to investigate the minimum thickness of semi-circular and elliptical rectangular arches, no work has been done to investigate the minimum thickness of skew arches. In this paper we computed the minimum thickness of semi-circular skewed masonry arches when subjected to their self-weight. Using the Discrete Element Method (DEM), a sensitivity study has been carried out to investigate the minimal barrel thickness with respect to the: a) angle of skew; b) construction method (false, helicoidal, and logarithmic); c) size of masonry units; and d) frictional resistance between masonry units. The construction method and the angle of skew significantly influences the minimum barrel thickness of the arch. For skew arches constructed using the false method, as the angle of skew increases, the minimum barrel thickness increases. However, for skew arches constructed using the helicoidal and logarithmic method, as the angle of skew increases, the minimum barrel thickness decreases. In contrast to rectangular arches, the size of the masonry units and the joint friction angle significantly influences the mechanical behaviour of skewed masonry arches.

1. Introduction

Masonry arch bridges constitute a significant proportion of European road and rail infrastructures. Most of them are well over 100 years old and support traffic loads many times above those originally envisaged. According to Orbán [33], there are approximately 200,000 masonry arch railway bridges in Europe. This is approximately 60% of the total bridge stock. Almost 70% of these masonry arch bridges are 100–150 years old, while 12% of them are older than 150 years. In addition, a proportion of masonry arch bridges span obstacles at an angle (or skew) other than 90°. This results in the faces of the arch not being perpendicular to its abutments and its plan view being a parallelogram (Fig. 1). Most of the masonry arches have been constructed with a small amount of skew (i.e. less than 45°), since those with large amount of skew present significant construction difficulties [28]. Different materials and methods of construction used in these bridges will influence their strength and stiffness. Although a great deal of work has been carried out to assess the strength of square or regular masonry arch bridges [18,17,34,28], comparatively little work has been undertaken to understand the behaviour of skew arches [20,42,37]. The analysis of skew arch bridges involves many difficulties and there is no universally accepted method of analysis yet. Today, in many countries, including UK, masonry skew arch bridges routinely assessed based on the assumption that they are rectangular in shape with an equivalent span of the skewed arch bridge (e.g. [3]). However, experience from current studies [20,39,37] demonstrated that this approach leads to conservative results, which is not representative of the actual strength and stiffness of the structure. Therefore, there is an increasing demand to understand the life expectancy of such bridges in order to inform maintenance, repair and strengthening strategies.

In recent years, sophisticated methods of analysis, like Finite Element Method (FEM), have been applied to understand the three dimensional behaviour of arches [5]. An overview of the different models performed in the 1990's can be found in Boothby [4] and Sarhosis et al. [39]. However, in such models, the description of the discontinuity is limited since they tend to focus on the continuity of the arch. Sophisticated FEM approaches (e.g. contact element techniques) are able to reflect the discrete nature of masonry. Examples of such models undertaken by Fanning and Boothby [11], Ford et al. [13] and Drosopoulos et al. [10]. The disadvantages of these methods are mainly associated with: a) high computational cost; b) inability to predict realistically the crack

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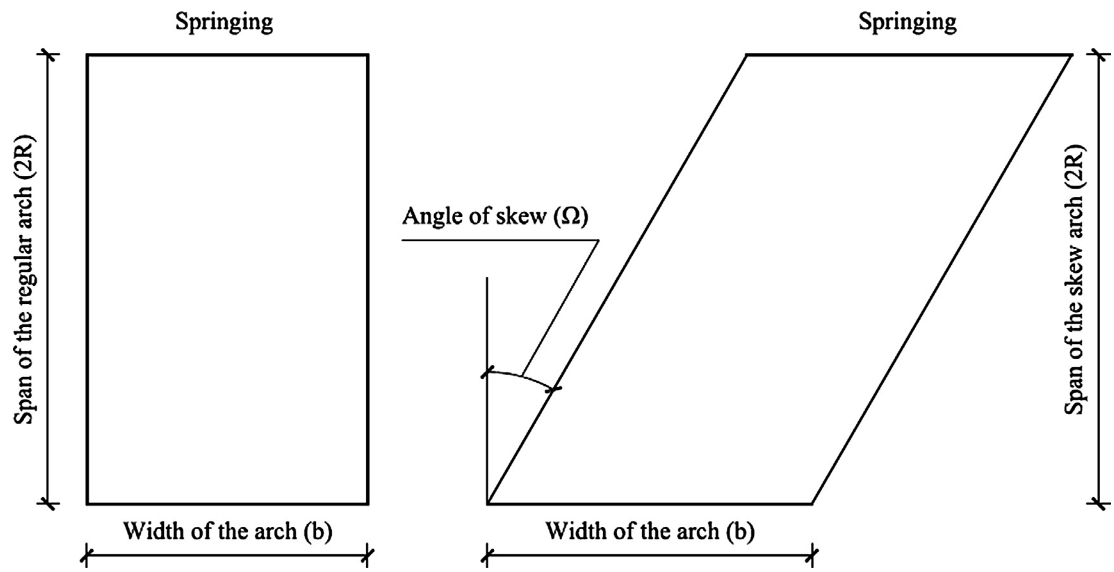

Fig. 1. Plan view of a regular and a skew arch (R is the corresponding radius of the mid-surface).

development at serviceability limit state; and c) convergence difficulties when blocks fall or slide excessively. An alternative and appealing approach is that represented by the Distinct Element Method (DEM), where the discrete nature of the masonry arch is truly incorporated. The advantage of the DEM is that it considers the arch as a collection of separate voussoirs able to slide and rotate relative to each other. The first version of DEM was developed by Cundall [8] to model blocky-rock systems and sliding along rock mass. The approach was later used to model masonry structures including arches [24,25,27,30,41,36,38], where failure occurs along mortar joints. These studies demonstrated that DEM is a suitable method to perform analysis of low bond strength masonry where failure is mainly at masonry unit-to-mortar interface [16].

Masonry arch bridges are composed of different structural components (e.g. piers, barrel, backfill, spandrel walls, parapets and wing walls) which interact each other. However, in order to understand the behaviour of masonry arch bridges, first it is of value to study each component separately and then move on and study their interaction. In this paper, use is made of the Discrete Element Method of analysis for the calculation of the minimum barrel thickness necessary for equilibrium of semi-circular masonry arches subjected to their own weight. In case of regular arches, the issue is settled: The purely rotational collapse mechanism that develops when the thickness of the arch is critically small have been investigated analytically and graphically by Milankovitch et al. [29] (see also [12] and found that forms a symmetric five-hinge mechanism just before collapse. However, up to now, no research work has been undertaken to investigate the minimum arch thickness of skew arches. Although the analysis of regular arches can be undertaken in two-dimensional space, the analysis of skew arches requires analysis in three-dimensional space. So, the three dimensional software 3DEC based on the Discrete Element Method (DEM) of analysis was used. Within the 3DEC model applied in the present study, each masonry unit of the arch is represented by a rigid element. Mortar joints are represented as zero thickness interface elements which can open and close according to the magnitude and direction of stresses applied to them. Also, a sensitivity study has been carried out to investigate the influence of the minimal barrel thickness with respect to the: a) angle of skew; b) construction method (e.g. false, helicoidal, and logarithmic method); c) size of masonry units; and d) frictional resistance between masonry units.

2. Constructional aspects of skewed masonry arch bridges

Masonry is strong in compression, but relatively weak in tension. Therefore, regular masonry arch bridges designed to be constantly under compression. To achieve this, the direction of forces within the arch should be normal to the coursing joints surface so that there will be no tendency in the successive courses to slide upon each other. The same idea is also adopted for the construction of masonry skew arches. In the 19th century, engineers, mathematicians and masons understood that for an arch to stand, the line of pressure should be parallel to the face of the arch. Hence, they positioned the voussoirs (e.g. stones, bricks) in such a way that the coursing joint surfaces should always be perpendicular to the face of the arch at every elevation. The other important factor considered for the construction of the skew arches related to the construction difficulties. Masons realised that construction was far easier when voussoirs had exactly the same size and were rectangular cuboid in shape. From the above observations, over the years, three main types of construction evolved for circular arches. These shown in Fig. 2:

a) *False skew arch:* This is the simplest form of construction where units are laid parallel to abutments (Fig. 2a).
b) *Helicoidal method (or English method):* In this method, the coursing joints are perpendicular to the face of the arch only at the crown. The coursing joints follow helix spirals. The advantage of this method is that each voussoir is similar in shape and size to all other voussoirs. However, for geometrical reasons and for the beds to remain parallel, the orientation of the block units causes the beds to "roll over" and thus rest on the springings at an angle. Gaps between masonry units in the arch usually filled with mortar (Fig. 2b).
c) *Logarithmic method:* In this method, the coursing joints are perpendicular to the face of the arch at all elevations. This is the most expensive method of construction since it requires varying sized masonry blocks and availability of high skilled masons, since almost every block in the arch barrel is of unique shape (Fig. 2c).