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Comprehensive Bayesian structural identification using temperature variation

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ABSTRACT

A modular Bayesian method is applied for structural identification of a reduced-scale aluminium bridge model subject to thermal loading. The deformation and temperature variations of the structure were measured using strain gauges and thermocouples. Feasibility of a practical, temperature-based, Bayesian structural identification is highlighted. This methodology used multiple responses to identify existent discrepancies of a model, calibrate the stiffness of the bridge support and establish uncertainty of a predicted response. Results show that the inference of a structural parameter is successful even in the presence of substantial modelling discrepancies, converging to its true physical value. However measurements should have a high dependency on the calibration parameters. Usage of temperature variations to perform structural identification is highlighted.

1. Introduction

The capability of a structural health monitoring (SHM) system to interpret monitored data is the main factor that dictates its performance and its usefulness to owners and local authorities.

Interpretation of the data using a physics-based model is advantageous because its development and usage as a predictive tool agrees with engineering knowledge, making it more understandable. However the main disadvantage is that the model has to be calibrated, before it can be used as a predictive tool. Moreover, using a deterministic model, i.e. a model where input parameters and response outputs are deterministic, rarely correlates well with real data, due to the complexity and inherent uncertainties of this latter one. Hence in most situations a probabilistic approach is more realistic [1] and methods such as model falsification [2], fuzzy numbers [3], Kalman filters [4], sampling methods [5], Markov processes [6] amongst others [7] have been developed for this purpose.

Bayesian inference is the basis of a class of well known methods which also allow to perform structural identification. Beck [8,9] is known as one of the pioneers of the application of Bayesian methods in SHM. Numerous research works have been conducted on the basis of this initial framework [10,11]. Based on this research, two fundamental problems might be thought as to why Bayesian methods were not widely applied in SHM practice. Firstly, the model parameters to be calibrated are often assumed as fixed physical properties of the infrastructure, while in reality these properties change due to external factors such as traffic and environmental variations [12], e.g. stiffness of the structure. Secondly, uncertainties due to modelling errors are only partially considered, despite being ubiquitous. They can be caused by: (a) discrepancy between the behaviour of a physics-based model and that of the real structure; and (b) numerical error in solving the partial differential equations (e.g. finite element method and mesh discretization). Component (a) is extremely difficult to quantify. Most of the present research [13,14] usually disregard this form of uncertainty or consider it as zero mean Gaussian distributed [6]. Only a limited number of authors in the SHM community, namely Higdon [15] and Simoen [16] have applied Bayesian methodologies under this scope. Model-based Bayesian structural identification with temperature variations is also considerably scarce in the literature [17,12].

Higdon applied a comprehensive modular Bayesian method originally developed by Kennedy and O'Hagan (KOH) [18,19], which was not widely accepted, presumably because of a lack of identifiability [20,21]. Identifiability is understood as the capability of inferring the true value of model parameters that represent a physical property, e.g. Young modulus, based on available data.

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Arendt et al. [22] suggested an improvement to the KOH original formulation to solve the identifiability problem, by using monitored data with diversified responses. This approach was validated on a simulated simply supported beam. We believe that this formulation is very comprehensive to quantify existent uncertainties and superior in some aspects to the ones used in previous works.

Based on this resurgent interest of the modular Bayesian method, the present work focus on its practical application for structural identification using thermal variations. Studies on advantages of using temperature loading for structural identification can be found in the works of Laory [23,24] and Yarnold [25]. The objective at hand is to test the performance of the improved algorithm on a scale aluminium bridge model subject to thermal loading. To the best of our knowledge, this test is the first practical application of this methodology, particularly for temperature-based structural identification.

Some of the advantages of using a scale model case study are: more realistic conditions, e.g. noise, inherent randomness and residual deformation of temperature loading; known structural parameters can be used to test the reliability of the methodology; possibility of easily testing different measurement scenarios; damaging the structure is permissible and allows to easily test damage identification methodologies.

This paper is organised as follows: In Section 2, a description of the model calibration formulation is given; Section 3 describes the aluminium bridge and its finite element (FE) model, presents a sensor placement analysis, application of the method and its results, and finally, Section 4 highlights the conclusions of the present work.

2. Model calibration formulation

This section describes the model calibration approach. A more detailed description can be found in [22]. An introduction to Gaussian process emulation is presented in Appendix A. An outline of the general formulation is given in the next subsection followed by a brief overview of the algorithm and of the numerical approach. To a more in depth description of the uncertainties considered by this methodology see Section 2 of [20].

2.1. Observation and numerical model equations

Let us now assume that a given continuous process ξ has n observations of q responses $\boldsymbol{Y}^e$ and is dependent on d design variables $\boldsymbol{X}^e$. Its observation equation can be written as

$$\boldsymbol{Y}^e(\boldsymbol{X}^e) = \xi(\boldsymbol{X}^e) + \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\varepsilon}^T = [\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_n]$ is an observation error that is assumed to follow a Gaussian distribution $\mathcal{N}(\boldsymbol{O}, \boldsymbol{\Lambda})$. On the other hand the unobservable process $\xi(\boldsymbol{X}^e)$ is described using a numerical model $\boldsymbol{Y}^m$ as follows

$$\xi(\boldsymbol{X}^e) = \boldsymbol{Y}^m(\boldsymbol{X}^e, \boldsymbol{\theta}^*) + \boldsymbol{\delta}(\boldsymbol{X}^e) \tag{2}$$

where $\boldsymbol{\delta}(\boldsymbol{X}^e)$ is a discrepancy function that translates the difference between the model and the true process, $\boldsymbol{Y}^m(\boldsymbol{X}^e, \boldsymbol{\theta}^*)$ is the model output and $\boldsymbol{\theta}^*$ are a r-dimensional vector of structural parameters. This equation is an ideal state of the model (i.e. the model is successfully calibrated) when the model parameters $\boldsymbol{\theta}$ take the values $\boldsymbol{\theta}^*$. Although our example updates only one parameter, the methodology can also consider multiple parameters, which is a common scenario in civil infrastructures [26,27,13].

It is important to mention that the discrepancy function is independent of the model output and is an unknown of the problem as well as the structural parameters. Now substituting equation number (2) in equation number (1) results in

$$\boldsymbol{Y}^e(\boldsymbol{X}^e) = \boldsymbol{Y}^m(\boldsymbol{X}^e, \boldsymbol{\theta}^*) + \boldsymbol{\delta}(\boldsymbol{X}^e) + \boldsymbol{\varepsilon} \tag{3}$$

which is the fundamental equation of the model calibration. Equation number (3) represents the process output to an input $\boldsymbol{X}^e$ within a domain of a calibrated status $\boldsymbol{\theta} = \boldsymbol{\theta}^*$, representing the best fit with the observed data.

The numerical model and the discrepancy function shall now be replaced by multiple response Gaussian processes (mrGp), whose hyperparameters have to be determined (see the Appendix for a definition and details). These hyperparameters characterise the mrGps and account for an approximation of its associated uncertainties, such as: variability of the numerical model; modelling discrepancies and observation error.

One way of determining the hyperparameters is by applying a Bayesian approach, which fully accounts for all the considered uncertainties and determines all the hyperparameters at the same time. However this implies a significant computational effort and is not recommended [28]. Instead a modular Bayesian approach shall be used, and is described in the following section.

2.2. Modular Bayesian approach

A modular Bayesian approach (MBA) separates the calibration process into four modules, on which the mrGp hyperparameters are estimated separately and progressively [29] as detailed in Fig. 1 (based on Fig. 5 from [20]).

Fixing the hyperparameters at an estimated value reduces the degree of approximation of the uncertainties covered by the mrGp. By doing so, the 'second order' effect of those uncertainties is being neglected. This means that preference has been given to recognise all of these sources of uncertainty, to a certain extent at a lower computational cost, rather than fully accounting for the uncertainties, with a considerable increase of computational effort.

This act of estimating and fixing the hyperparameters is applied progressively, when moving from module 1 to module 2 and from module 2 to module 3. Estimation is done with numerical optimisation methods by maximising the likelihood between the mrGp and the available data.

In our case we used a MATLAB genetic algorithm (GA) routine. An initial population of size 40 was generated in the [0–1] range, with default values for Gaussian mutation function (mean 0, standard deviation 1 and shrinkage of the standard deviation as generations go by 1) and scattered crossover function (0.8 fraction of the population at the each next generation). Convergence criteria are set as either a maximum number of 100 generations or an average change in the fitness value less than 1×10^{-6}.

It is important to stress that the discrepancy function is not being updated i.e. it is not the same as a GA fitness function. Instead the GA in module 2 (see Fig. 1) aims to estimate the parameters of a statistical model (a Gaussian process), that approximates the discrepancy function. This is done through maximum likelihood estimation (MLE), which implies that the fitness function of the GA is the likelihood function.

In module 3, Bayes' theorem is used for approximating the posterior distribution of θ. In contrast to other approaches mentioned in the introduction, its likelihood function contains the two mrGp approximated in modules 1 and 2, now with its hyperparameters fixed.

3. Aluminium bridge subjected to thermal loading

In this case-study a reduced-scale laboratory aluminium bridge inspired by the New Joban Line Arakawa (Japan) railway bridge, was built at the Warwick Civil Engineering Laboratory and subjected to thermal loading due to infrared heaters. Typical daily ambient temperature in the laboratory ranged from 291.15 K up