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Analytical solution for free vibrations of rotating cylindrical shells having free boundary conditions

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ABSTRACT

In this paper free vibrations of rotating cylindrical shells with both ends free are studied. The model used also allows for considering a flexible foundation supporting the shell in the sense of a radial and circumferential distributed stiffness. Furthermore, a circumferential tension (hoop stress) which may be due to pressurisation or centrifugal forces is taken into account. Natural frequencies and mode shapes are determined exactly for both stationary shells and for shells rotating with a constant angular speed around the cylinder axis. Trigonometric functions are assumed for the circumferential mode shape profiles, and a sum of eight weighted exponential functions is assumed for the axial mode shape profiles. The functional form of the axial profiles is shown to greatly vary with the roots of a characteristic bi-quartic polynomial that occurs in the process of satisfying the equations of motion. In the previously published work it has been very often assumed that the roots are two real, two imaginary, and two pairs of complex conjugates. In the present study, a total of eight types of roots are shown to determine the whole set of mode shapes, either for stationary or for rotating shells. The results using the developed analytical model are compared with results of experimental studies and very good agreement is obtained. Also, a parametric study is carried out where effects of the elastic foundation stiffnesses and the rotation speed are examined.

1. Introduction

Dynamics of shells have been an active research topic for well over a century. Some early works dating from the 19th century [1–5], were followed by the developments in the 20th century [6–19]. Many geometries occurring in various engineering structures can be seen as shells. Among these, circular cylindrical shells form a particular class. It is often the case that a cylindrical shell spins around its axis, which makes its dynamic behaviour more complex. Rotating shell structures are found in engineering applications such as rotor systems of gas turbine engines, high-speed centrifugal separators, rotating satellite structures, and automotive tires, to name a few.

Early studies on rotating cylinders include the work of Bryan [20] who studied vibrations of a rotating ring and described the travelling modes phenomenon. Di Taranto and Lessen [21], and also Srinivasan and Lauterbach [22] studied Coriolis and centrifugal effects on infinitely long rotating cylindrical shells. Zohar and Aboudi [23], and also Saito and Endo [24] presented such investigations on finite long rotating cylinders. Endo et al. [25] performed an experimental study of flexural vibration of a thin rotating ring. Padovan [26] studied the free vibration of rotating cylinders subjected to pre-stress. Kim and Bolton also considered the effects of rotation on the dynamics of a circular cylindrical shell [27]. They suggested that the model may be used to predict the characteristics of a rotating tire after performing a kinematic compensation on the results of a stationary tire analysis. Huang and Soedel [28] used the nonlinear strain displacement relationships of Herrmann and Armenakas [29] and the corresponding set of equations of motion for a spinning shell in the co-rotating reference frame. The authors have solved the free and forced vibration problem assuming simply supported boundary conditions (the so-called shear diaphragm boundary conditions). This is a favourable type of a boundary condition from a mathematical point of view. This is because mode shape axial profiles do not exhibit a change of their functional form. Thus simple sine or cosine functions of the axial coordinate may be used [18,19,27–30]. The natural frequencies can be calculated as roots of a characteristic polynomial, which

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was shown to be bi-cubic if the shell does not rotate. In case the shell spins at a constant speed, also the odd coefficients of the polynomial occur. Thus in case of a non-rotating cylindrical shell this bi-cubic polynomial has three pairs of roots where each pair consists of a positive and a negative natural frequency having the same absolute value. Physically this underlies the existence of the backward and forward rotating modes that superimpose into "regular" vibration modes in case the cylinder is stationary. With spinning cylinders the six natural frequencies have distinct absolute values, and thus the rotating modes occur [28]. The three pairs of positive and negative frequencies correspond to three types of modes, which could be named bending modes, longitudinal modes, and shear modes. This classification is based on whether the radial, axial, or circumferential displacement component is the most prominent in a particular vibration pattern.

In general, short expressions for calculating natural frequencies of either rotating or stationary cylindrical shells are not possible if no further simplifications or assumptions are made to reduce the order of the characteristic polynomial [17,19,31–33]. For example, the Donnel-Mushtari-Vlasov equation can yield a reasonably short closed form expression for natural frequencies of a non-rotating cylindrical shell provided that simply supported boundary conditions are assumed [19].

However, in case of other boundary conditions the situation complicates. Consequently a number of studies have also been dedicated to vibration of cylindrical shells with other types of simple boundary conditions [34–37]. For example, Chung expressed the displacements as product of Fourier series for the axial modal displacements and trigonometric functions for the circumferential modal displacements. The author used Stokes' transformation to obtain expressions for derivatives of the Fourier series [37]. Boundary conditions such as free-free, clamped-free and clamped-clamped are considered in the study. This methodology has been recently extended by Sun et al. [38] to rotating cylindrical shells including the effects of centrifugal and Coriolis forces and the initial hoop tension. Alternatively, the Rayleigh–Ritz method can be employed to derive the frequency equations of rotating cylinders. Utilising the Rayleigh–Ritz method, Sun et al. [39] took the characteristic orthogonal polynomial series as the admissible functions with classical homogeneous boundary conditions, or with more general boundary conditions, by utilising artificial springs to simulate the elastic constraints imposed.

An exact approach to deal with other types of boundary conditions has been used by Warburton [36]. He analysed the free vibration problem using Flügge equations and considered either both ends clamped or both ends free of a non-rotating cylindrical shell. A number of mode shapes and natural frequencies were calculated in [36] by assuming identical boundary conditions at the two ends of the shell, and analysing separately symmetric and anti-symmetric modes. The author considered the case where the roots of the characteristic polynomial are of a particular form: two real, two imaginary and four complex.

A complete analytical solution for free vibrations of a circular cylindrical shell of finite length, supported by an elastic foundation, having both ends free, either stationary or rotating, is given in this paper. The equations of motion are based on the strain-displacement relationships of Hermann and Armenakas [29]. It is shown that it is necessary to consider eight types of roots of the characteristic polynomial in order to derive eight types of mode shapes. All types of modes may occur with both non-rotating and spinning shells having free ends. For each mode shape type the free-free boundary conditions are satisfied exactly. In order for the boundary conditions to be satisfied, the determinant of the boundary condition matrix must vanish. This fact is used to determine the natural frequencies of both stationary and rotating shells.

The paper is structured into five sections. The mathematical model is developed in the second section. The free vibrations of an example rotating shell are discussed in the third section. The third section also contains a comparison of the analytical results to results of different experimental studies. The fourth section is dedicated to a parametric study where the effect on the natural frequencies of different parameters of the model is studied. Appendix to the paper contains various coefficients needed to shorten the main expressions in the paper expressed as a function of the material and geometrical shell parameters.

2. Mathematical model

The rotating cylindrical shell is shown schematically in Fig. 1.

Assuming the free vibration problem the equations of motion are [28,29]:

$$\begin{bmatrix} \mathsf{L}_{x,u}+\rho h\frac{\partial^2}{\partial t^2} & \mathsf{L}_{x,v} & \mathsf{L}_{x,w} \\ \mathsf{L}_{\phi,u} & \mathsf{L}_{\phi,v}+\rho h\left(\frac{\partial^2}{\partial t^2}-\Omega^2\right) & \mathsf{L}_{\phi,w}+2\rho h\Omega\frac{\partial}{\partial t} \\ \mathsf{L}_{z,u} & \mathsf{L}_{z,v}-2\rho h\Omega\frac{\partial}{\partial t} & \mathsf{L}_{z,w}+\rho h\left(\frac{\partial^2}{\partial t^2}-\Omega^2\right) \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \mathbf{0}. \tag{1}$$

The linear operators $\mathsf{L}_{x,u}$, $\mathsf{L}_{x,v}$, $\mathsf{L}_{x,w}$, $\mathsf{L}_{\phi,u}$, $\mathsf{L}_{\phi,v}$, $\mathsf{L}_{\phi,w}$ and $\mathsf{L}_{z,u}$, $\mathsf{L}_{z,v}$, $\mathsf{L}_{z,w}$ are:

$$\mathsf{L}_{x,u} = \frac{(\mu-1)K - 2\,N_{\phi,i}}{2a^2}\frac{\partial^2}{\partial\phi^2} - (K+N_{x,i})\frac{\partial^2}{\partial x^2}, \tag{2}$$

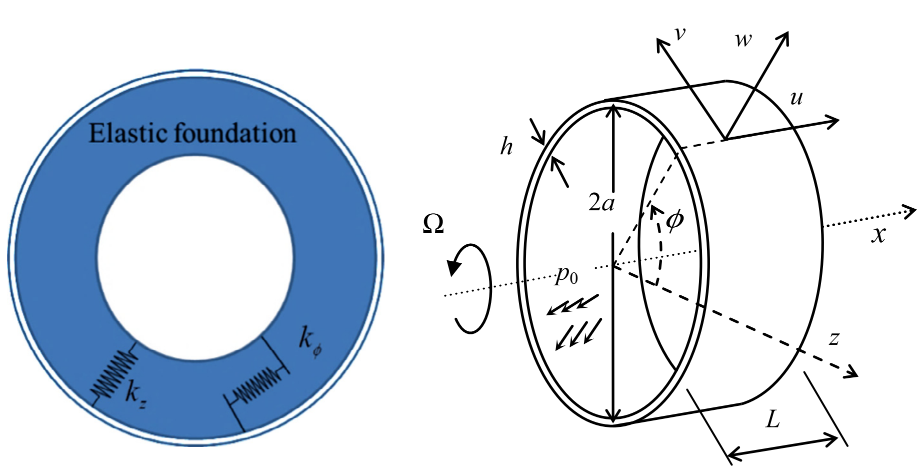

Fig. 1. The rotating cylindrical shell.