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Linear buckling of quadrangular and kagome gridshells: A comparative assessment

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ABSTRACT

The design of gridshells is subject to strong mechanical and fabrication constraints, which remain largely unexplored for non-regular patterns. The aim of this article is to compare the structural performance of two kind of gridshells. The first one is the kagome gridshell and it is derived from a non-regular pattern constituted of triangles and hexagons. The second one results from a regular pattern of quadrangles unbraced by diagonal elements. A method is proposed to cover kagome gridshells with planar facets, which reduces considerably the cost of fabrication of the cladding.

The sensitivity of kagome gridshells to geometrical imperfections is discussed. The linearised buckling load of kagome gridshells is then compared to the one of quadrilateral gridshells. The most relevant design variables are considered in the parametric study. Two building typologies are studied for symmetrical and non-symmetrical load cases: dome and barrel vault. It reveals that the kagome gridshell outperforms quadrilateral gridshell for a very similar construction cost.

1. Introduction

Grid-shells are structures made of beam elements that act as continuous shells structures. The choice of a grid pattern influences both fabrication and structural behaviour. For example, triangulated structures are known to be stiffer than quadrangular meshes. Quadrangular grishells rely on the bending stiffness of connections, whereas triangulated gridshells benefit from a shell-like behaviour without the need for rigid connections. The better structural performance of triangular gridshells is however at the cost of an increased node complexity due to higher node valence. In quadrangular meshes, panels are however not necessarily planar, and only specific curve networks on surfaces or shape-generation strategies guarantee meshing with planar quadrilaterals [1–4]. There is thus a necessary trade-off between design freedom and fabrication constraints.

This article focuses on a lesser known family of pattern, called kagome grid pattern, composed from triangles and hexagons and represented in Fig. 1. The kagome pattern can be found in Japanese basketry, where the members are woven. We focus here on applications to structural engineering and consider non-woven pattern, where all the neutral axes of the beams are concomitant, and the beams are rigorously straight. Like quadrilateral grids, kagome grids present a node valence of four, which indicates a reasonable cost of fabrication. Among other usage, kagome grids have been used in the architecture of Shigeru Ban and for ornamentation purpose. Their structural possibilities remain however largely unexplored, and little is known on the planarity of the facets, a key element to the economy of the envelope.

Kagome grid pattern and quadrilateral grid pattern have the same node valence, and their structural behaviour can be compared qualitatively. Rigid connections are necessary to guarantee in-plane shear stiffness of these patterns. However, their relatively low node valence assures the existence of a large families of torsion-free beam offsets compatible with the use of deep beams [5]. Kagome and quadrilateral grid patterns can thus be built with very similar technological solutions. Their relative structural performances is however not quantified and will be studied in this paper, whose main contributions are:

- a strategy for the covering of kagome meshes with planar facets, demonstrating that they could be a viable alternative to triangular or quadrilateral meshes;

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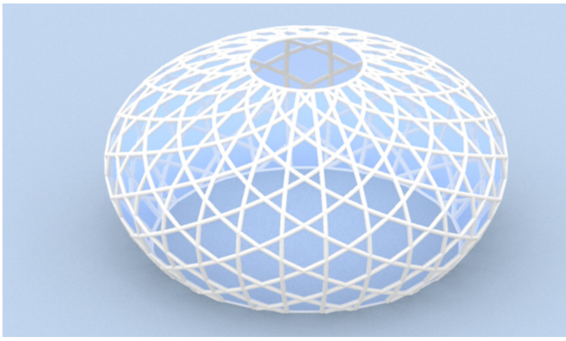

Fig. 1. A kagome grid pattern covered with planar facets generated with the method described in this paper.

- a parametric study comparing the linear buckling load of kagome gridshells with quadrangular gridshells for shapes covered with planar facets;
- design guidelines for kagome gridshells.

The article is organised as follows: the first section presents the motivations for this work as well as relevant literature in the field of mechanics of gridshells. The second section introduces the methodology chosen to assess the structural behaviour of kagome gridshells. The third section gathers the results of the conducted parametric study. A brief discussion and conclusion sum up the contributions of the present work.

1.1. Previous work on the mechanics of single-layered lattice shells

The structural behaviour of gridshells is usually governed by non-linear effects, most noticeably buckling [6]. Four buckling configurations can be observed in gridshells:

- Global buckling in the manner of a shell;
- Member buckling;
- Snap-through of one node;
- In-plane rotation of one node.

Some design recommendations, often emphasizing simple shapes, like spherical cupolas have been published. Gioncu published a state of the art on the buckling of reticulated structures in 1995 [7]. A report produced by the Working Group of the International Association for Shells and Spatial Structures (IASS) in 2005 completes this review with analytical and numerical results, demonstrating the important advances made in that field [8]. A novel issue is to be published in 2016. A design guide for the stability of reticulated shells with a thorough literature review is proposed in [9], showing a great mastery of this topic.

These guidelines identify two approaches to evaluate the structural behaviour of a grid structure: homogenisation methods and numerical experiments. This article establishes a parametric numerical study, and uses previous work on homogenisation of grid structures to comment the numerical results.

1.1.1. Homogenisation and equivalent shell thickness

Homogenisation techniques aim to formulate an equivalent continuous behaviour of a heterogeneous structure with a cell repeated periodically. These methods use the superposition principle and usually work well for structures with a linear behaviour [10]. They have been successfully used for planar grids [11], but a rigorous extension to gridshells is difficult because of the loss of periodicity, due to the variations of curvature. A discussion on this topic is proposed by Gioncu and Balut [12].

The advantage of equivalent thickness model is that they provide structural engineers with simpler formulas and can be of practical interest for conceptual structural design. Some attempts to provide equivalent shell thickness have been used in previous studies [13–16]. However, these models do not allow for the modelling of localised buckling and the study of the influence of imperfections for shell structures remains tedious for non-trivial shapes. Nowadays, the ever-growing computational power makes the use of finite element modelling and non-linear analysis ubiquitous in practice, and numerical simulations are often preferred to homogenisation formulæ.

1.1.2. Numerical experiments

Numerical methods are used for the practical design of gridshells, because they allow for integration of complex issues, like material nonlinearities or geometrical imperfections. Some guidelines for the analysis of reticulated domes have been proposed by Kato et al. [17,18]: these studies introduced geometrical imperfections and semi-rigid nodes. Bulenda and Knippers [6] performed parametric studies on domes and barrels vaults and evaluated the influence of imperfections on the stability of gridshells. A more complete study using finite element analysis to evaluate local node stiffness of patented connections has been performed by Huang et al. [19]. Bruno et al. assessed the influence of nodal imperfection and of Eigenmode Imperfection Method (EIM) more recently [20]. Malek et al. [15] performed numerical investigations on the buckling of spherical cap domes and considered geometrical values, like grid spacing, or height over span ratio, as parameters. This approach lead to recommendations for the design of gridshells with triangular or quadrangular layout.

Other studies have evaluated the influence of residual stresses in elastic gridshells [16,21]. A more complete analysis was performed on the elastic gridshell built for the Soliday's festival in Paris, considering accidental ruin of some members [22]. These studies show that high bending stresses due to the form-finding process of elastic gridshells have little influence on the buckling capacity of domes. Such procedures could be extended to steel structures, in order to assess the influence of other residual stress fields on the stability of gridshells.

1.2. Imperfections

There are many differences between the ideal numerical shell models and the built structures. These differences, or imperfections can be of different nature: loads, geometry, material, residual stresses in the members. Thin shells are known to be sensitive to imperfections [23]. These parameters are often set as a global geometrical imperfections. Gioncu and Balut also point out that geometrical imperfection tend to govern over material nonlinearities for large span structures [12].

Typically, the difference between the built geometry and the computed model is of a few centimeters at most [24]. It is therefore necessary to introduce a norm, in order to asses realistic imperfections. In the following, the norm $\|\cdot\|_\infty$ defined as the maximal displacement is used. Bulenda and Knippers propose a higher bound of $L/500$ for the imperfection with the infinity norm [6]. Based on data on the precision requirements for built project [25], Malek et al. studied an imperfection of 3 mm [15].

The choice of the shape function is discussed in Section 2.4. The first buckling mode is recommended by design codes, and was used for example for the design of the roof of the British Museum and the Palacio de Comunicaciones [26,25]. However, different studies show that other imperfections shapes should