Fusion Engineering and Design 114 (2017) 91–96

Contents lists available at ScienceDirect

Fusion Engineering and Design

journal homepage: www.elsevier.com/locate/fusengdes

Investigation of natural frequencies of laser inertial confinement fusion capsules using resonant ultrasound spectroscopy

Xiaojun Ma[a,b], Xing Tang[b], Zongwei Wang[b], Qian Chen[c], Menglu Qian[c], Jie Meng[b], Yongjian Tang[a,b], Yaming Zou[a], Hao Shen[a], Dangzhong Gao[b,*]

[a] Institute of Modern Physics, Fudan University, Shanghai 200433, China
[b] Research Center of Laser Fusion, CAEP, Mianyang 621900, China
[c] Institute of Acoustic, Tongji University, Shanghai 200433, China

HIGHLIGHTS

- The frequency equation of isotropic multi-layer hollow spheres was derived using three-dimension (3D) elasticity theory and transfer matrix method.
- The natural frequencies of the capsules with a millimeter-sized diameter are determined experimentally using resonant ultrasound spectrum (RUS) system.
- The predicted natural frequencies of the frequency equation accord well with the observed results.
- The theoretical and experimental investigation has proved the potential applicability of RUS to both metallic and non-metallic capsules.

ARTICLE INFO

Article history:
Received 15 May 2016
Received in revised form
13 November 2016
Accepted 6 December 2016
Available online 12 December 2016

Keywords:
Inertial confinement fusion
Resonant ultrasound spectroscopy
Natural frequency
Capsule

ABSTRACT

The natural frequency problem of laser inertial confinement fusion (ICF) capsules is one of the basic problems for determining non-destructively the elasticity modulus of each layer material using resonant ultrasound spectroscopy (RUS). In this paper, the frequency equation of isotropic one-layer hollow spheres was derived using three dimension (3D) elasticity theory and some simplified frequency equations were discussed under axisymmetric and spherical symmetry conditions. The corresponding equation of isotropic multi-layer hollow spheres was given employing transfer matrix method. To confirm the validity of the frequency equation and explore the feasibility of RUS for characterizing the ICF capsules, three representative capsules with a millimeter-sized diameter were determined by piezoelectric-based resonant ultrasound spectroscopy (PZT-RUS) and laser-based resonant ultrasound spectroscopy (LRUS) techniques. On the basis of both theoretical and experimental results, it is proved that the calculated and measured natural frequencies are accurate enough for determining the ICF capsules.

1. Introduction

In laser inertial confinement fusion (ICF) experiments, a spherical capsule, which is made from glass, polymer, beryllium or high-density carbon, is employed as the thermonuclear reaction container. A representative capsule for SG-III Facility in China is shown in Fig. 1. The capsule is an isotropic three-layered hollow microsphere filled with deuterium-tritium (D-T) fuel gases. The outer diameter and total wall thickness of capsule is about 1000 μm and about 100 μm, respectively. The material of each layer is polystyrene (PS), polyvinyl alcohol (PVA) and glow discharge polymer (CH), respectively.

In order to achieve the required implosion efficiency, there are many specifications that need to be fully characterized for the capsules, such as geometrical structures, sphericity, outer and inner surface roughness, dopant concentration, impurity level and deuterium-tritium (D-T) fuel content [1–4]. Increasing evidences show that the elastic modulus of material is a key physical quantity for understanding the physical nature of material and controlling their mechanic and dynamic properties [5]. Consequently, it is of

* Corresponding author.
E-mail address: dgaocn@163.com (D. Gao).

http://dx.doi.org/10.1016/j.fusengdes.2016.12.007

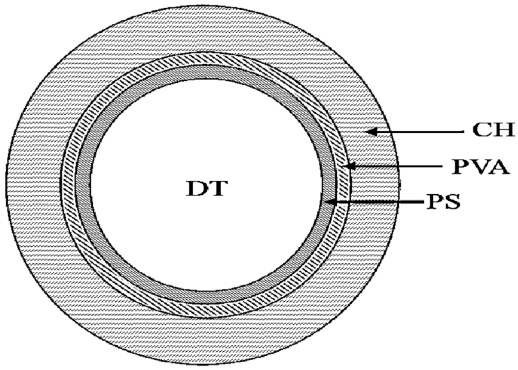

Fig. 1. The diagram of a representative capsule for SG Facility.

critical importance to measure accurately the elastic modulus of ICF capsule for controlling the quality of capsule. Due to the complicated structure of ICF capsule, the accurate and nondestructive determination of elastic modulus of each layer material in a measurement is considered as one of the most challenging problems in ICF target characterization.

Over the past 20 years, resonant ultrasound spectroscopy (RUS) has been widely applied to determine the elastic modulus, surface defects and residual stress of sample with known geometries, such as solid sphere, solid cylinder or rectangular parallelepiped [6–12]. The natural frequencies of ICF capsules also have been used to determine the internal gas density and calculate the lowest even-order cavity boundary perturbation amplitudes [13,14]. However, little attention has been given to evaluate the elastic properties of the layered spherical shells both in the theory and experiment. There are two remaining issues that need to be solved for charactering the elastic modulus of ICF capsule using RUS technique, the first one is to derive theoretically the frequency equation of ICF capsule vibrations, and the second one is to measure accurately the natural frequencies of ICF capsule.

The purpose of this article is to establish the theoretical and experimental basis for determining the elastic modulus of ICF capsule using RUS method. To solve the issues above-mentioned, the frequency equation of an isotropic multiplayer spherical shell was derived firstly. Then, the natural frequencies of representative capsules with a millimeter-sized diameter were measured with a combined RUS apparatus. Lastly, we discussed the strengths of RUS techniques for ICF capsule characterization and potential application.

2. Natural frequencies of isotropic hollow spheres

2.1. Analysis of a one-layer hollow sphere

Owing to the widely applications of elastic spherical shells in various engineering fields, the free vibrations problem has attracted increasing attentions and become one of the basic problems in elastodynamics. The corresponding problem of an isotropic spherical shell was first investigated by Lamb as early as 1890s [15]. Shah and his coworker solved the characteristic frequency equation of an isotropic hollow single-layer sphere by three-dimensional (3D) elasticity theory and made a numerical comparison between the 3D elasticity theory and shell theory [16,17]. The radial vibrations of an isotropic two-layer hollow sphere, which is independent of variables θ and ϕ in the spherical coordinate, was considered by Yehuda Stavsky and J. Barry Greenberg [18]. W.Q. Chen studied the free vibrations of an isotropic multi-layer hollow sphere using a state-space method [19]. In the following section, we should discuss the vibration problem of an isotropic multilayer hollow sphere by 3D elasticity theory and transfer matrix method.

For an isotropic elastic sphere, the displacement vector u in spherical coordinates (r, θ, ϕ) can be written by a scalar potential ϕ, vector potentials ϕ_1 and ϕ_2[20]

$$u = \nabla\phi + \nabla \times (r\phi_1 + \nabla \times (r\phi_2)) \tag{1}$$

Where$\nabla$is the usual del operator, ϕ,ϕ_1 and ϕ_2 satisfy the wave equations. If there is a harmonic dependence on time, the following equations can be obtained

$$\begin{aligned} &\nabla^2\phi + {k_1}^2\phi = 0 \\ &\nabla^2\phi_1 + {k_2}^2\phi_1 = 0 \\ &\nabla^2\phi_2 + {k_2}^2\phi_2 = 0 \end{aligned} \tag{2}$$

Where$k_1$andk_2 are the wave vector of longitudinal and transverse wave, respectively.

The stress-displacement relation:

$$\begin{aligned} &\sigma_{rr} = \lambda\varepsilon + 2\mu\frac{\partial u_r}{\partial r} \\ &\sigma_{r\theta} = \mu\left(\frac{\partial u_\theta}{\partial r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r}\right) \\ &\sigma_{r\phi} = \mu\left(\frac{\partial u_\phi}{\partial r} + \frac{1}{r\sin\theta}\left(\frac{\partial u_r}{\partial \phi} - u_\phi \sin\theta\right)\right) \\ &\varepsilon = \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} + \frac{1}{r\sin\theta}\left(\frac{\partial u_\phi}{\partial \phi} + u_r\sin\theta + u_\theta\cos\theta\right) \end{aligned} \tag{3}$$

Where$\lambda$and μ are Lame's constants.

Using separation variable technique, the solutions to Eq. (2) are given by

$$\begin{aligned} &\phi = (Aj_n(k_1r) + Bn_n(k_1r))P_n^m(\cos\theta)\begin{pmatrix}\cos m\phi \\ \sin m\phi\end{pmatrix} \\ &\phi_1 = (Cj_n(k_2r) + Dn_n(k_2r))P_n^m(\cos\theta)\begin{pmatrix}\cos m\phi \\ \sin m\phi\end{pmatrix} \\ &\phi_2 = (Ej_n(k_2r) + Fn_n(k_2r))P_n^m(\cos\theta)\begin{pmatrix}\cos m\phi \\ \sin m\phi\end{pmatrix} \end{aligned} \tag{4}$$

Where$j_n(kr)$,$n_n(kr)$are spherical Bessel functions of the first and second kinds, respectively. Symbols "$\begin{pmatrix}\cos m\phi \\ \sin m\phi\end{pmatrix}$" means "$\cos m\phi$" or "$\sin m\phi$". A, B, C, D, E *and* F are arbitrary constants.

Substituting Eq. (4) into Eq. (1), the displacement vector can be written as

$$\begin{aligned} &u_{r,1} = \left(A\frac{dj_n(k_1r)}{dr} + B\frac{dn_n(k_1r)}{dr}\right)P_n^m(\cos\theta)\begin{pmatrix}\cos m\phi \\ \\ \sin m\phi\end{pmatrix} \\ &u_{\theta,1} = \frac{(Aj_n(k_1r) + Bn_n(k_1r))}{r}\frac{dP_n^m(\cos\theta)}{d\theta}\begin{pmatrix}\cos m\phi \\ \\ \sin m\phi\end{pmatrix} \\ &u_{\phi,1} = \frac{m(Aj_n(k_1r) + Bn_n(k_1r))}{r}\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{pmatrix}\cos m\phi \\ \\ -\sin m\phi\end{pmatrix} \end{aligned} \tag{5a}$$