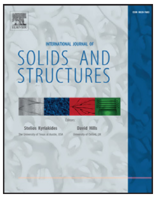
International Journal of Solids and Structures 000 (2017) 1–12

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr



Non-dissipative structural evolutions in granular materials within the small strain range

Mehdi Pouragha, Richard Wan*

Civil Engineering Department, University of Calgary, 2500 University Dr NW, Calgary, AB, Canada T2N 1N4

ARTICLE INFO

Article history:
Received 20 September 2016
Revised 13 January 2017
Available online xxx

Keywords:
Granular materials
Micromechanics
Non-dissipative
Microstructure
Fabric anisotropy
Small strains

ABSTRACT

Microscale mechanisms involving the loss and gain of contacts in granular materials are non-dissipative in nature, but yet irreversible. This is because the creation or disintegration of contacts as an instantaneous event does not pertain to any global deformation of the granular system. Such a contact loss and gain regime is intriguing and has also been shown to play a significant role in altering the internal structure of granular materials, even at the relatively small strain range where irreversibilities can exist within deformation domains often attributed to elasticity. The current study offers a coherent decomposition of mechanisms affecting the microstructure of granular media, and subsequently investigates the contribution of non-dissipative/irreversible mechanisms to global structural rearrangement. An analytical scheme is put forward that relates the directional variations in contact losses and gains to the probability distributions of contact forces and interparticle separating distance, respectively. Thus, microstructural evolution can be statistically computed in terms of two key microvariables, i.e. coordination number and fabric anisotropy, and verified through 2D Discrete Element Method (DEM) simulations. The analytical scheme presented here provides an accurate description of microvariable evolution laws that are needed to formulate micromechanical constitutive models for granular materials.

1. Introduction

Granular materials display distinctive behavioural characteristics that are tied to their ability of readapting their internal structure in response to external loads via the loss or gain of contacts. In such a process, various concurrent mechanisms all different in nature are triggered at the microscopic scale resulting into substantial changes in contact density and directional distribution, even within the small strain range (Kruyt, 2012; Pouragha and Wan, 2016). In fact, such an evolving internal structure has been recognised as the main source of non-linearity in granular material behaviour (Nicot and Darve, 2006; Agnolin and Roux, 2007). Accordingly, this implies that the formulation of any micromechanical model for granular materials must be predicated on the principle of an evolving interparticle contact and force network with associated microvariables that have yet to be described (Rothenburg et al., 1989; Agnolin and Roux, 2007).

The choice of coordination number and fabric anisotropy as being the proper microvariables has been broadly embraced in the literature. As a recall of basic definitions, coordination number herein refers to the average number of contacts per particle, while fabric anisotropy relates to the directional distribution of contact normals. However, the way these micro-parameters evolve with respect to external loading is not yet well understood (Jenkins and Strack, 1993; Kruyt, 2012; Azéma et al., 2009; Bathurst and Rothenburg, 1990; Radjai et al., 2012). For instance, Discrete Element Method (DEM) simulation results reveal that when a densely packed granular assembly is subjected to deviatoric loading, the contact structure initially evolves following a non-dissipative mechanism. The latter is associated to small deformations with yet, a considerable increase in fabric anisotropy and decrease in coordination number (Pouragha and Wan, 2016; 2017) as a result of contact gains and losses. This non-dissipative regime is shown to persist until a threshold coordination number is reached beyond which limit, dissipative mechanisms take over through contact sliding/rolling to govern the internal structure evolution (Pouragha and Wan, 2016).

From the above-mentioned observations, it turns out that this initial regime of contact gains and losses is the principal cause for microstructural changes, especially in the small strain region ($\sim 10^{-3}$) which is traditionally considered to be within the so-called elastic domain. This phenomenon is illustrated in Fig. 1 for a typical biaxial DEM simulation on an initially dense assembly, which is in accordance with previous findings in Kruyt (2012). The zone $A-B$ in Fig. 1 refers to the axial strain range below 10^{-3}. It turns

---

* Corresponding author.
E-mail address: wan@ucalgary.ca (R. Wan).

http://dx.doi.org/10.1016/j.ijsolstr.2017.01.039
0020-7683/© 2017 Published by Elsevier Ltd.

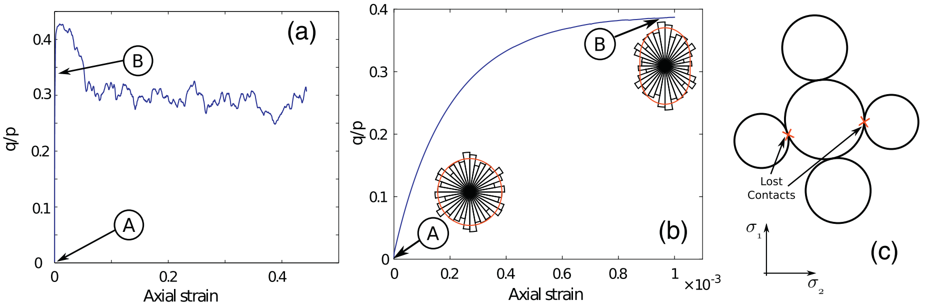


Fig. 1. (a) Typical DEM simulation results of a 2D biaxial test. (b) Initial part of stress–strain curve (axial strain below 10^{-3}) where the contact network is shown to become considerably anisotropic in a small strain range. (c) Schematic drawing of initially dominant contact loss regime along minor principal stress direction.

out that ignoring the non-linear effects arising from such a structural evolution, e.g. the change in coordination number and fabric anisotropy, can potentially lead to significant errors, e.g. in soil dynamics, where simple elastic models are often adopted for modelling sand in the small strain regime (Agnolin and Roux, 2008; Roux, 2015). These errors can also become considerable when interpreting geophysical test data in granular media where wave propagation characteristics can be directly influenced by contact density and fabric anisotropy (Agnolin and Roux, 2007; Goddard, 1990; Gudehus, 2006; Ostrovsky and Johnson, 2001; Somfai et al., 2005).

It is noted further that the initial evolution of granular microstructure in the small strain range is often ignored, even in micromechanically based models. The common belief is that any elastic response of granular materials by definition precludes structural changes (Kruyt et al., 2010; Chang et al., 1995; Cambou et al., 2004). However, this assumption overlooks the possibility of non-dissipative mechanisms whose existence within a hypothetical elastic zone is legitimate and would not raise any contradiction in theory. Based on this argument, the irreversible responses in granular materials should not be associated entirely to dissipative mechanisms (i.e. plastic strains), and, in theory, one can imagine granular assemblies, similar to those with artificially high friction, whose response is purely energy-conserving but irreversible. This undermines the existence of an energy potential from which elastic strain can be derived (Agnolin and Roux, 2007). To prevent any confusion, it should be clarified here that we are not espousing the idea that an exclusive elastic zone exists for granular materials. The authors' previous studies (Pouragha and Wan, 2016) clearly show that any attempt to model the small strain behaviour of granular materials behaviour based on pure elasticity naturally involves approximations that need to be justified in practice.

While the relevance of contact gain and loss regime has been recognised in the granular material literature (Kruyt, 2012; Radjai et al., 2012) the number of studies on the analytical modelling of these mechanisms are limited. One of the first such attempts can be found in Jenkins and Strack (1993) where a simplified theoretical framework based on the homogeneous strain assumption is put forward in order to model the contact loss and gain regime. Further interpretations of fabric evolution through geometrical constraints can be found in the works of Radjai and coworkers (Radjai et al., 2012).

With these motivations in perspective, the current study investigates the origin of non-dissipative mechanisms in granular materials and their role in the evolution of microstructure in comparison to other prominent mechanisms. Two-dimensional biaxial DEM simulations have been performed and the contributions of different microscopic mechanisms to internal structure evolution have been extracted. The results show a substantial reorganization of internal granular structure due to non-dissipative mechanisms even in the small strain range. In line with our previous findings (Pouragha and Wan, 2016), it is shown that non-dissipative mechanisms dominate the microstructural changes in the initial stages of loading until a threshold for contact network resiliency is reached where dissipative mechanism takes over the microstructural evolution.

Furthermore, an analytical scheme is put forward where the contact loss and gain regime can be computed in terms of the realignment of force network with external loads. Given the interparticle force and distance distributions, the change in coordination number is related to the net amount of contact number change, while the directional distribution of contact gain/loss regime determines the change in contact fabric anisotropy. Knowing the evolution of these two key parameters, statistical expressions for stress can then be employed to compute the evolution of other parameters such as force anisotropies. Comparisons with DEM simulation results confirm that the proposed calculation scheme is capable of predicting the evolution of microstructure of granular materials in a statistical sense within an acceptable degree of accuracy.

2. Microscopic mechanisms

The mechanisms existing at the particle scale in a granular assembly can be categorised, based on their nature, into three broad groups as follows:

1. *Non-dissipative/reversible* (e): This category refers to changes in interparticle forces which induce recoverable contact deformations. For a deformable granular assembly, an elastic change in contact force can slightly alter the position of particles with respect to each other. However, for stiff enough contacts, the effect of these mechanisms on the contact configuration can be ignored.
2. *Dissipative/irreversible* (p): The main mechanism in this category is the frictional rolling/sliding[1] at contact points which is prominent in frictional granular assemblies. This mechanism affects the internal contact structure since the direction of contact normal can considerably change due to sliding. The evolution of the contact network due to these mechanisms and its relation to kinematics of granular materials has been previously studied by the authors, see Wan and Pouragha (2014) and Pouragha and Wan (2017).
3. *Non-dissipative/irreversible* (δ): This category includes instantaneous events such as contact loss and gain. While there is no deformation or strain attributed to these phenomena, they can substantially alter the contact structure. The net sum of contact gain and loss regime characterises the coordination number change and the directional distribution of the changes determines the change in fabric anisotropy.

---

[1] The two mechanisms "rolling" and "sliding" are grouped into one mechanism as they always coexist.

Please cite this article as: M. Pouragha, R. Wan, Non-dissipative structural evolutions in granular materials within the small strain range, International Journal of Solids and Structures (2017), http://dx.doi.org/10.1016/j.ijsolstr.2017.01.039