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Minimum energy based method to predict the multiple cracking pattern in quasi-brittle beam

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ABSTRACT

A minimum energy based method is proposed to predict the evolution of multiple cracking pattern in a pure bending beam made of quasi-brittle material. The minimum energy is found from the energy landscape of the cracked beam calculated by the Finite Element (FE) Method, and the crack pattern corresponding to the minimum energy is taken as the crack pattern most likely to occur. The fracture behavior of the quasi-brittle material is simulated by cohesive zone model (CZM) using the bilinear tension softening law. After appropriate normalization based on dimensional analysis, the multiple cracking process is found to be governed by only three parameters and their effects are systematically studied. Physically, the ratio between the “crack-tip toughness” and the “bridging toughness” is important for the crack pattern evolution. As a validation, the proposed method is used to predict the crack pattern of fiber reinforced concrete beam and good agreement with experiment is achieved. The goal of this paper is twofold. First, with a systematic parametric study focusing on the constitutive law of quasi-brittle material, this investigation would facilitate material design or selection for crack control; Second, by quantitatively determining the crack pattern development during the whole multiple cracking process based on a sound physical principle, i.e., the commonly adopted minimum energy criterion, the result can serve as a benchmark for checking the reliability of other methods for multiple cracking analysis.

1. Introduction

Multiple cracking phenomena are widely found both in nature and engineering practice, such as in the formation of basalt column (Goehring et al., 2009), desiccation of clay soil (Tang et al., 2008), fracking of gas or oil bearing rock (Bažant et al., 2014), thermal shock of ceramic or glass (Bahr et al., 1986; Geyer and Nemat-Nasser, 1982; Jiang et al., 2012), shrinkage of concrete pavements (Shah and Weiss, 2006) as well as in the bending of reinforced concrete beams. During the multiple cracking process, complex crack patterns form due to the cracks interacting with each other and instability problem usually occurs. An example of crack pattern evolution under thermal shock, which introduces a time-varying temperature gradient perpendicular to the surface of a semi-infinite domain, is illustrated in Fig. 1. The prediction of crack patterns has been extensively studied, both theoretically and numerically, mainly for brittle materials. Assuming a periodic crack pattern, existing theoretical investigations are conducted in two different ways: one is to determine the critical state at which only alternative cracks continue propagating, based on the sign change of second derivatives of the total energy with respect to crack length, e.g., Nemat-Nasser et al. (1978), Bažant et al. (1979) and Sumi et al. (1980); the other is to find the configuration corresponding to the minimal energy by evaluating the energy associated with different assumed crack patterns, e.g., Jenkins (2005). Without adopting the assumption of periodic crack pattern, the multiple cracking process can also be simulated directly using numerical methods. For example, Li et al. (2013) proposed a nonlocal failure model to simulate the evolution of crack pattern in ceramic during quenching; Tang et al. (2016) and Xu et al. (2016) analyzed the same problem using statistical mesoscopic damage models; Bourdin et al. (2014) studied this problem with gradient damage model under the variational framework. The above models are all implemented with the finite element method. Other numerical methods such as atomistic method (Jagla and Rojo, 2002) and boundary element method (Bahr et al., 1993) have also been used to study the multiple cracking problem.

However, very few studies have been conducted on the multiple cracking problem of quasi-brittle materials such as clay, concrete or fiber reinforced concrete, in which the process zone is relative large (Anderson, 2005). Nonlinear fracture models, e.g., smeared crack model by Bažant and Oh (1983) or fictitious crack

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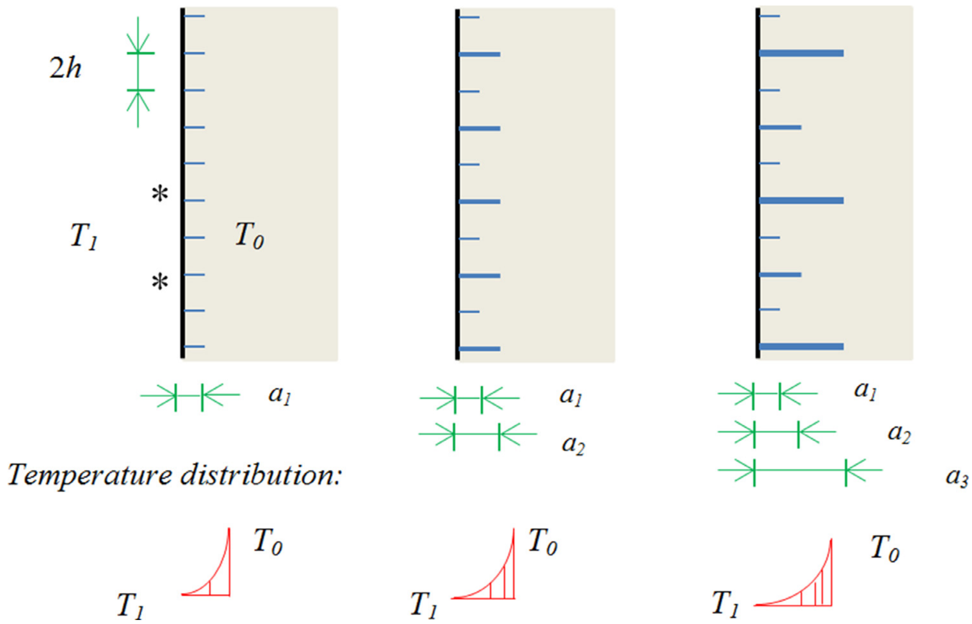

Fig. 1. Typical evolution of multiple cracking pattern (after Bažant et al., 1979).

model by Hillerborg et al., (1976), need to be used for such materials to study their multiple cracking behavior. Horii and Inoue (1997) studied the pattern evolution of multiple cracks using smeared crack model and judged the instability by the sign change of second derivative of total energy with respect to the crack opening width. They showed that the consideration of instability is important for capturing the ultimate strength and ductility of the beam. Shi et al. (2004) analyzed multiple cracking problem of concrete beam adopting the discrete crack model. They proposed a scheme that only one crack is active in each trial calculation and selected the case corresponding to the lowest load. Amarasiri and Kodikara (2014) studied thermal shock crack using discrete crack model and discussed how the characteristic length affects the multiple cracking behavior.

However, it is noted that in Horii and Inoue (1997) and Shi et al. (2004), little attention was paid to the detailed crack pattern evolution process and the softening law they used is representative of plain concrete, which is relatively brittle and multiple cracking problem is not very important. Also, the minimal load criterion in Shi et al. (2004) lacks a sound theoretical basis and is seldom referred to in the literature. Amarasiri and Kodikara (2014) did not consider the instability condition, which is very important in multiple cracking process. Therefore, there is a need for systematic studies to better understand and reliably predict the multiple cracking process in quasi-brittle materials.

To address the above research gaps, a minimum energy based method is proposed to study the multiple cracking problem in quasi-brittle materials. The fracture behavior of quasi-brittle materials is described by bilinear softening law, of which the parameters are systematically discussed, covering both relatively brittle and relatively tough materials. Based on the widely accepted minimum energy criterion, the crack pattern evolution during the whole multiple cracking process can be quantitatively predicted and has a sound physical basis. The method is illustrated with a beam under pure bending, which is one of the most basic loading configurations. The pure bending condition can be found in the middle zone of a 4 point bending test, which is widely used to study the structural behavior of materials. In the following sections of the paper, the basic concept of the energy landscape and the minimum energy criterion, as well as their application to multiple cracking problem is first introduced. Based on this methodology, a simplified FE model containing only one crack is created and analyzed using the FE software ABAQUS to evaluate the energy of cracked beam. Based on the results of FE analysis, a scheme is further developed to construct the energy landscape and to find the crack pattern corresponding to the minimum energy. After detailed discussion of the result of a particular case, a systematical parametric study is performed, providing insights to material design or selection for crack control. Finally the proposed energy based approach is used to determine the crack pattern in a fiber reinforced concrete beam, and comparison with test results is carried out to verify its validity.

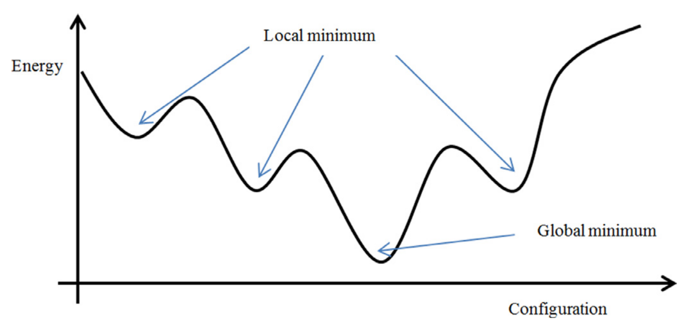

Fig. 2. Schematic of energy landscape.

2. Methodology

2.1. Basic concept of energy landscape and minimum energy

The concept of “Energy Landscape” is widely used in physics, chemistry and biology (Wales, 2003), and refers to the mapping of the configuration of complex systems, e.g, molecules, and their corresponding energy levels. According to the particular problem studied, the energy considered may be the Gibbs (or Helmholtz) free energy or the potential energy. A typical energy landscape is illustrated in Fig. 2. As shown in this figure, there are local and global minimum points corresponding to different equilibrium configurations. At other points, the difference in energy may drive the system to move continuously to neighboring states with lower