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Planar dynamics of large-deformation rods under moving loads

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ABSTRACT

We formulate the problem of a slender structure (a rod) undergoing large deformation under the action of a moving mass or load motivated by inspection robots crawling along bridge cables or high-voltage power lines. The rod is described by means of geometrically exact Cosserat theory which allows for arbitrary planar flexural, extensional and shear deformations. The equations of motion are discretised using the generalised-α method. The formulation is shown to handle the discontinuities of the problem well. Application of the method to a cable and an arch problem reveals interesting nonlinear phenomena. For the cable problem we find that large deformations have a resonance detuning effect on cable dynamics. The problem also offers a compelling illustration of the Timoshenko paradox. For the arch problem we find a stabilising (delay) effect on the in-plane collapse of the arch, with failure suppressed entirely at sufficiently high speed.

1. Introduction

The problem of a continuously distributed system carrying a moving concentrated mass has broad applications in mechanics and engineering, including space tethers, satellite antennas, launch systems, robotic arms [1], cranes [2], flexible manipulators [3], high-speed train railroads and highway bridges with moving vehicles [4,5].

The classical example of a moving-mass problem is the idealisation of a vehicle-bridge system. In this case the moving vehicle is usually treated as a moving force, or load, of constant magnitude, while the bridge is modelled, for instance, as a simply-supported beam. This problem is therefore more accurately described as a moving-load problem. The moving load assumption does not take into account the inertial forces of the moving mass and the effect of the beam on the mass. For an overview of the sizeable early literature on the vibration theory of moving-load problems we refer to Fryba's monograph [6]. More recent works on moving loads or masses travelling along beams are [7–9], while more specific studies include moving loads or masses along curved beams or arches [10–12], inclined beams [13], multi-span beams [14,15] and tapered beams [16]. Meanwhile, loads or masses travelling along cables (modelled as strings, i.e., without bending stiffness), are studied in Refs. [17–19]. See also the

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review in Ref. [20].

All the above works restrict themselves to beams or cables undergoing small deflections (in Ref. [8] moderately large deflections are considered) and all consider only two-dimensional bending vibrations (in Ref. [19] equations are first developed for three-dimensional deformations but these are then condensed to a planar model). With the current drive to use thinner and lighter materials, in order to save material and reduce costs, large deformations become increasingly important.

In this paper we formulate the moving-load problem for a geometrically-exact Cosserat rod that can undergo arbitrary two-dimensional flexural, extensional and shear deformations. This planar case seems to cover most moving-load applications. The restriction to the moving-load problem is justified by the fact that in cases with large deflections the speeds are likely to be relatively small so that inertial effects can be neglected. As a typical application we are here thinking of inspection robots crawling along bridge cables or high-voltage power lines [21]. We leave the proper treatment of the moving mass problem, with inertia included, for a future publication.

Usually in the literature when (moderately) large deformations are considered, approximate equations are derived (usually involving a geometrically nonlinear strain-displacement relation [8]). These approximate equations are often arguably more complicated (and less transparent) than the geometrically exact equations. Moreover, these equations then still need to be solved numerically, typically by employing a Galerkin expansion (using on the order of 10 terms) [8,19]. These Galerkin expansions are well known to suffer from lack of uniform convergence (Gibbs phenomenon) in problems with jump discontinuities, as occur in the internal force in moving-load problems [22,23].

Purely numerical methods using time-stepping algorithms directly on the equations of motion without initial approximation is often avoided because of convergence limitations [24]. However, sophisticated current numerical methods can now solve the exact nonlinear equations with little difficulty and this is the approach taken in this paper.

Cosserat theory describes the evolution of a material director frame, attached to the cross-section of the rod, as it moves along the rod and in time. By introducing an angle to parametrise the director frame we obtain a formulation free from kinematic constraints (and from Euler-angle singularities). In this reduced description it is natural to include the centreline integration within the full discretisation. This gives a more efficient and accurate scheme than existing three-dimensional formulations that use post-processing (i.e., updating of the rotation matrix, typically by using the Rodrigues formula, to compute the directors, and integrating the tangent vector, typically by using the trapezoidal rule) to obtain rod shapes [25,26].

As is natural with large deformations, we are not only interested in natural modes of vibration but in transient dynamics of large amplitude. We therefore discretise the equations of motion using the generalised-α method in both the spatial and temporal domain. This numerical method is found to have good convergence and stability properties for moving-load problems. Of course our approach can also be applied to cables and rods with fixed attached masses. Point loads or masses greatly complicate the description of cable motion because of singularity problems in internal forces [18,22–24].

The paper is organised as follows. In Section 2 we introduce the planar formulation of Cosserat rod dynamics, while the numerical discretisation is presented in Section 3. In Section 4 we then apply the theory to a few planer problems. The first of these is a 'pendulum test' in which we drop a hinged rod under gravity from a horizontal position for a sequence of bending stiffnesses approaching the rigid pendulum limit. This problem is also used to verify energy conservation of the numerical scheme to second-order accuracy. We then apply our method to both a cable and an arch problem. We find interesting new nonlinear behaviour induced by moving loads on flexible structures. For the cable problem we find that large deformations have a resonance detuning effect. For the arch problem we find the moving load to have a stabilising effect. Some similar stabilisation effect of the speed of the load (a decrease of the midspan displacement of a cable under an increase of the speed of a moving mass) is noted with surprise in Ref. [19]. This behaviour is also observed in Ref. [16]. Both these results are for small deflections. In our present case, with large deflections, the effect is much stronger and we find that an arch that would collapse under a given quasi-static load will stand if it is traversed by the same load moving at sufficiently high speed. Conclusions follow in Section 5.

2. Formulation of Cosserat rod dynamics

In Cosserat theory the configuration of a rod deforming in the plane (by arbitrary bending, shear and extension, but not torsion) is determined by two vector-valued functions $(\mathbf{r}(s,t), \mathbf{d}_1(s,t)) \in \mathbb{R}^2 \times \mathbb{R}^2$ of arclength $s \in [0, L]$ of the unstressed rod and time t (L being the length of the unstressed rod). Here $\mathbf{r}$ represents the centreline of the rod, while $\mathbf{d}_1$ is a unit vector in the plane of the deformed rod pointing along a material line in the rod's cross-section. We also introduce the unit normal vector to the cross-section at s, $\mathbf{d}_2$, and parametrise the director (or body) frame $\{\mathbf{d}_1, \mathbf{d}_2\}$ by means of the angle θ as follows

$$\mathbf{d}_1 = \cos\theta\ \mathbf{i} + \sin\theta\ \mathbf{j}, \tag{1}$$

$$\mathbf{d}_2 = -\sin\theta\ \mathbf{i} + \cos\theta\ \mathbf{j}, \tag{2}$$

relative to the fixed inertial frame $\{\mathbf{i}, \mathbf{j}\}$ (see Fig. 1). Note that since the rod is shearable the director $\mathbf{d}_2$ will in general not be equal to the tangent vector, $\mathbf{r}'(s,t) := \partial \mathrm{r}(\mathrm{s},\mathrm{t})/\partial \mathrm{s} =: \partial_s \mathrm{r}(\mathrm{s},\mathrm{t})$, to the rod. In fact, for this tangent vector we will write

$$\mathbf{r}' = \mathbf{v}, \tag{3}$$

with components v_1 and v_2 in the body frame: $\mathbf{v} = v_1\ \mathbf{d}_1 + v_2\ \mathbf{d}_2$. (We shall generally use subscripts '1' and '2' to indicate components of any vector in the body frame and subscripts 'x' and 'y' to indicate components in the fixed frame $\{\mathbf{i}, \mathbf{j}\}$.) v_1 and $v_2 - 1$ are the shear and extensional strains in the rod, respectively.