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The wave attenuation mechanism of the periodic local resonant metamaterial

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ABSTRACT

This research discusses the wave propagation behavior and attenuation mechanism of the elastic metamaterial with locally resonant sub-structure. The dispersion relation of the single resonance system, i.e., periodic spring mass system with sub-structure, could be derived based on lattice dynamics and the band gap could be easily identified. The dynamically equivalent properties, i.e., mass and elastic property, of the single resonance system are derived and found to be frequency dependent. Negative effective properties are found in the vicinity of the local resonance. It is examined whether the band gap always coincides with the frequency range of negative effective properties. The wave attenuation mechanism and the characteristic dynamic behavior of the elastic metamaterial are also studied from the energy point of view. From the analysis, it is clarified that the coupled Bragg-resonance band gap is much wider than the narrow-banded local resonance and the corresponding effective material properties at band gap could be either positive or negative. However, the band gap is totally overlapping with the frequency range of negative effective properties for the metamaterial with band gap purely caused by local resonance. The presented analysis can be extended to other forms of elastic metamaterials involving periodic resonator structures.

1. Introduction

Metamaterial is generally regarded as man-made materials with microstructures that would exhibit unusual physical properties, which could not be observed in natural materials. A lot of researches focus on the dynamically equivalent negative material property of metamaterials in last few years. These negative effective properties are demonstrated by metamaterials based on locally vibrational resonant structures to control wave prorogation. For applications, it is common that the propagating wavelength is much larger than the characteristic size and the periodic length of the resonant structures. Sometimes, the spatial periodicity of the resonant structure is not necessarily required. Interesting phenomena, such as negative refraction, self-collimation of wave, become possible through the implementation of metamaterial with negative effective properties.

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Double negative elastic metamaterial, analogy to Veselago's medium in electromagnetism, was demonstrated by several research groups [1–5]. The negative material properties of metamaterial are commonly achieved through incorporation of resonators. Li and Chan [1] realized both negative effective density and bulk modulus by suspending soft rubber spheres in water. Ding et al. [2] suggested a double unit structure, with one specially designed to have very strong monopolar resonance while the other one to have very strong dipolar resonance, to implement double negative metamaterial. Single negative acoustic metamaterial were introduced by Liu and Sheng [6]. They introduced localized resonant structures, i.e., lead balls coated with silicone rubber, embedded inside epoxy to get negative effective modulus. Fang et al. [7] adopted Helmholtz resonators to create ultrasonic metamaterial and observed effective negative bulk modulus both numerically and experimentally. Huang and Sun [8] proposed sub-structured periodic spring mass system and investigated the wave attenuation mechanism in elastic metamaterial with negative effective mass density near resonance. Later, Huang and Sun [9] suggested a multi-resonator elastic metamaterial for effective wave attenuation and their focus has limited to the emergence and overlapping of bandgaps for wave filtering application. The elastic metamaterial with extreme Young's modulus was designed with local resonator and the capability of the metamaterial to selectively block or filter unwanted waves was numerically demonstrated [10]. It is concluded by the authors that the effective Young's modulus would become negative in a certain frequency range which coincides with the band gap.

Since the resonator is commonly incorporated in the metamaterial design, the local resonance tends to cause narrow band gap and results in the equivalent negative material properties. However, the band gap involving periodic structures could also be caused by other mechanisms, such as Bragg scattering at wavelength compatible to the periodic length. Hence, some researchers [11–17] proposed metamaterials by exploiting the band gaps caused by both Bragg scattering and local resonance for wave filters or blocker at various frequencies. A complete overview of historical developments and an in-depth literature review of recent progress in the field with special consideration given to aspects concerning the fundamentals of dynamics, vibrations, and acoustics can be found in literature [18]. The discussion of the metamaterial coupling local resonance with Bragg band gaps usually focused on widening the band gap. However, negative effective material properties are a relatively new concept proposed in elastic metamaterial research, especially useful in the promising application of double negative metamaterial with negative refractive index. The discussion in literature [8,10] suggested the total overlapping frequency range of negative material properties and band gap, which is somehow misleading. One purpose of this paper is point out the fact that the effective material properties at band gap could be either positive or negative depending on the band gap mechanism. The periodic spring mass model with sub-structure proposed in the literature [8] will be employed. The wave attenuation mechanism and the characteristic dynamic behavior of the acoustic metamaterial inside the band gap will be investigated numerically. Attention will focus on the mechanisms that prevent harmonic waves from propagating in the metamaterial when the effective property is either positive or negative. The other purpose is to investigate the possibility of creating wider band gap in the metamaterial involving periodic resonators for the wave blocking or filtering application.

2. Model and methods

The wave propagation behavior inside the periodic mass spring system with sub-structure as illustrated in Fig. 1, hereafter called “single resonance system”, will be studied. The single resonance system indicates only one sub-structure, that would cause local resonance, within one unit cell in order to differentiate from multi-resonator case discussed in the literature [9]. One unit cell of the single resonance system is composed of two masses, i.e., m_1, m_2 and two springs, i.e., k_1, k_2. The main structure is consisting of repeated mass m_1 and spring, elastic constant k_1, connected with each other at periodic length L. The sub-structure, mass m_2 and spring of elastic constant k_2, is connected to mass m_1 of the main structure.

Lattice dynamics method will be employed to solve for the wave propagation inside the system. The equations of motion for the masses in the jth unit cell can be written respectively as

$$m_1\frac{d^2u_1^{(j)}}{dt^2}=k_1\left(u_1^{(j+1)}-u_1^{(j)}\right)-k_1\left(u_1^{(j)}-u_1^{(j-1)}\right)+k_2\left(u_2^{(j)}-u_1^{(j)}\right) \tag{1}$$

$$m_2\frac{d^2u_2^{(j)}}{dt^2}=-k_2\left(u_2^{(j)}-u_1^{(j)}\right) \tag{2}$$

where $u_{1,2}^{(j)}$ denotes the displacements of the masses in the jth unit cell. Assume the displacement of the jth mass, $u_{1,2}^{(j)}$, changes with time as $e^{(-i\omega t)}$, therefore

$$\frac{d^2u_{1,2}^{(j)}}{dt^2}=-\omega^2u_{1,2}^{(j)} \tag{3}$$

It is assumed the wave propagates with wavelength much larger than the periodic length, L, of the single resonance system. As a result, the displacement fields in the jth unit cell can be expressed as