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Trapped modes and resonance wave transmission in a plate with a system of notches

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ABSTRACT

The paper deals with guided wave propagation and diffraction in an elastic plate with two and more parallel grooves (notches). It is a continuation of the preceding studies of a single-notch diffraction focused on the manifestation of the trapped mode and resonance transmission effects in the case of several obstacles. Those effects occur at the frequencies close to the nearly real scattering resonance poles, and the forms of wave localization near the notches are predetermined by the eigenforms associated with these poles. Therefore, the change of location of nearly real poles with additional notches and its dependence on their spacing are the issues of prime concern. The increase in number and assembling in groups of resonance poles and transmission peaks, which was known for multi-obstacle guides of other nature, have also been revealed and experimentally confirmed. The theoretical research is conducted on the basis of low-cost semi-analytical models, and the numerical simulation is experimentally validated through the laser Doppler vibrometry.

1. Introduction

The present work is a continuation of preceding papers [1,2] on the case of guided wave (GW) propagation in plates with several grooves. The introduction of systems of obstacles into the waveguide structures is a common approach to the creation of artificial structures with special wave properties. Most often they are made using periodic systems of obstacles, e.g., interdigital contacts or grooves used in acoustoelectronic devices [3], quantum systems [4,5], phononic lattices and crystals [6–8], and elastic guides with alternating cells [9] or point-wise defects [10].

A deep groove (notch) commonly screens traveling waves, reflecting most of the wave energy, so that the incident wave packet $\mathbf{u}_0$ bounces from such an obstacle. However, at certain frequencies, the incident wave is not reflected but captured and localized at the notch, exhibiting prolonged oscillations with a gradual wave energy re-emission in both directions [1,2]. This phenomenon is a typical example of the trapped mode effect [11,12]. Mathematically, the trapped modes are eigensolutions associated with real and nearly real spectral points of the corresponding time-harmonic diffraction problem, i.e. with the natural scattering resonance frequencies f_n located in the complex frequency plane f at and near the real axis.

With different kinds of obstacles, the resonance poles f_n manifest themselves in different ways. For example, a resonance GW interaction with a horizontal crack (delamination) in an elastic plate results in a sharp blocking of the time-averaged energy transfer in the steady-state time-harmonic wave field at the frequencies $f = \mathrm{Re} f_n$ while the crack may hardly affect the wave propagation at frequencies outside of their vicinity [13]. In this case, a frequency plot of the transmission coefficient $\kappa^+ = E^+/E_0$

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is featured by sharp thin dips at $f = \mathrm{Re} f_n$; here E^+ is the time-averaged wave energy transmitted by GWs behind the obstacle and E_0 is the incident wave energy.

On the other hand, with a system of obstacles, the presence of a nearly real natural frequency f_n in a gap band unblocks the guide in its vicinity [14]. At the curve $\kappa^+(f)$, it looks like a sharp peak indicating resonance GW transmission at $f = \mathrm{Re} f_n$. Since the resonance poles are located along the real axis with a certain regularity, it yields a typical comb-like or rippled pattern of the transmission coefficients $\kappa^+(f)$ intrinsic to guides of various nature [4–6,9,10,14–16]. Physically, this phenomenon results from the localized modes associated with the Fabry-Perot interference between two reflecting obstacles. The transmission peaks could also be explained by the eigenmodes of a finite-length specimen cut out from the infinite waveguide that contains the obstacles [14,16].

The poles f_n, and so the transmission peaks, are assembled in local successive groups, and the number of the poles or peaks in each group is $M - 1$, where M is the number of obstacles. With two obstacles, every group contains just one pole with one corresponding transmission peak in a comb-like pattern. Adding the same equally spaced obstacles does not change the frequency ranges, beneath which the groups of resonance poles are located in the complex frequency plane, but results in peaks' splitting into $M - 1$ thinner subpeaks.

From this point of view, a deep single notch considered in the preceding papers manifests itself as two obstacles yielding a comb-like pattern of resonance transmission (e.g., Fig. 2 in Ref. [2], as compared with the typical patterns for a simple string waveguide with point-wise defects in Fig. 3 taken from Ref. [10]). The two obstacles, providing the resonance localization of oscillation between them, are the vertical sides of the notch while a comparatively thin bottom crosspiece is a resonant wave accumulator and conductor.

Such a mechanism of wave energy capturing by a notch-like obstacle with its further resonance transmission has been revealed through analytically-based simulation and experimentally confirmed at the predicted frequencies through laser Doppler vibrometer (LDV) measurements [2]. With two and more notches, the number of segments between the shielding GWs butts increases, and specific manifestation of the trapped mode and resonance transmission effects becomes more complicated. Therefore, the present paper addresses the following issues that arise with a system of notches:

- the change of location of resonance poles with additional notches and how it depends on the notch spacing;
- assembling of the poles in groups and the corresponding increase in number of transmission peaks as it takes place with several obstacles in the guides of other nature;
- specific forms of wave localization (trapped mode eigenforms) corresponding to different kinds of resonance poles;
- specific features of transient wave capturing and re-emission.

2. Mathematical and experimental frameworks

The basic mathematical model selected from the developed 1D and 2D models for the routine low-cost simulation of GW propagation in multiply notched plates (Figs. 1 and 2) is the system of Timoshenko plate equations [17].

$$\begin{aligned} &Yu'' + \rho\omega^2 u = 0,\\ &\mu\chi(w'' - \varphi) + \rho\omega^2 w = 0,\\ &YI\varphi'' + \mu h\chi(w' - \varphi) + I\rho\omega^2\varphi = 0, \end{aligned} \tag{1}$$

where the prime denotes derivatives with respect to the horizontal coordinate x.

The unknown functions $u(x)$, $w(x)$ and $\varphi(x)$ specify the complex displacement amplitude $\mathbf{u} = (u_x, u_z)$ of the 2D time-harmonic oscillation $\mathbf{u}e^{-i\omega t}$ by the relations

$$\begin{aligned} &u_x = u(x) - \xi\varphi(x),\\ &u_z = w(x). \end{aligned} \tag{2}$$

Here Y and μ are Young's and shear moduli, ρ is density, h is plate's thickness, $I = h^3/12$ is the reduced moment of inertia, $\chi = \pi^2/12$ is the shear correction factor, ξ is the local transverse coordinate reckoned from the central axis of the plate, and $\omega = 2\pi f$ is the circle frequency.

The thickness h in Eq. (1) is different for different segments D_n: $x_{n-1} \le x \le x_n$ ($x_0 = -\infty, x_N = \infty$), on which the notch faces (dock points) x_n divide the M-notched strip domain (Fig. 1). Eq. (1) are set in every of the $N = 2M + 1$ subdomains D_n with their specific values of the input geometric and material parameters h_n, Y_n, ρ_n and μ_n. Therefore, in general, the model simulates low-

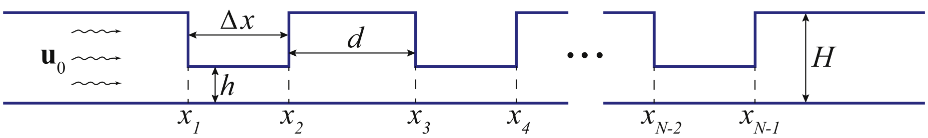

Fig. 1. Geometry of the problem: elastic strip with M equally spaced notches.