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On optimization of energy harvesting from base-excited vibration

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ABSTRACT

This paper re-examines and clarifies the long-believed optimization conditions of electromagnetic and piezoelectric energy harvesting from base-excited vibration. In terms of electromagnetic energy harvesting, it is typically believed that the maximum power is achieved when the excitation frequency and electrical damping equal the natural frequency and mechanical damping of the mechanical system respectively. We will show that this optimization condition is only valid when the acceleration amplitude of base excitation is constant and an approximation for small mechanical damping when the excitation displacement amplitude is constant. To this end, a two-variable optimization analysis, involving the normalized excitation frequency and electrical damping ratio, is performed to derive the exact optimization condition of each case. When the excitation displacement amplitude is constant, we analytically show that, in contrast to the long-believed optimization condition, the optimal excitation frequency and electrical damping are always larger than the natural frequency and mechanical damping ratio respectively. In particular, when the mechanical damping ratio exceeds a critical value, the optimization condition is no longer valid. Instead, the average power generally increases as the excitation frequency and electrical damping ratio increase. Furthermore, the optimization analysis is extended to consider parasitic electrical losses, which also shows different results when compared with existing literature. When the excitation acceleration amplitude is constant, on the other hand, the exact optimization condition is identical to the long-believed one. In terms of piezoelectric energy harvesting, it is commonly believed that the optimal power efficiency is achieved when the excitation and the short or open circuit frequency of the harvester are equal. Via a similar two-variable optimization analysis, we analytically show that the optimal excitation frequency depends on the mechanical damping ratio and does not equal the short or open circuit frequency. Finally, the optimal excitation frequencies and resistive loads are derived in closed-form.

1. Introduction

Energy harvesting from ambient vibrations has become a promising solution to alternative energy for many applications, such as wireless sensors [1], microelectromechanical systems (MEMS) [2], portable electrical devices [3,4], and some large-scale on-site power generation [5,6]. Among different energy harvesting topologies, one of the most popular consists of a single degree-of-freedom (SDOF) spring-mass-damper system coupled with energy harvesting circuitry via electromagnetic or piezoelectric transducers [1,2,7–16].

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Since the mechanical and electrical system are coupled via the transducer, the mechanical and electrical parameters have mutual effects on energy extraction. In order to optimize the extracted energy, parameters in both systems have to be simultaneously considered. To this end, many efforts have been made to determine the optimal mechanical and electrical parameters of electromagnetic energy harvesting [1,7,12,14,16], all of which basically reached the same conclusion that the maximum power output of the SDOF vibration energy harvester is achieved when the excitation frequency equals the natural frequency of the mechanical system and when the electrical damping due to the energy harvesting circuitry is the same as the mechanical damping. However, this conclusion, followed by a substantial body of literature [2,8,10,15], is based on a two-stage process in which the mechanical and electrical parameters are optimized in two sequential stages. In the first stage, power into the mechanical system is optimized by matching the natural frequency with the excitation frequency (frequency matching), followed by the second stage in which power into the electrical system is optimized by matching the impedance of the electrical load (impedance matching). For example, in Williams and Yates' work [8], the mechanical system is considered to be at resonance before the electrical impedance is subsequently matched. Furthermore, in the work of Roundy et al. [1], it is assumed that the excitation and natural frequency are equal before deriving the optimal resistive load of the maximum power. Later, in Stephen's work [14] the natural frequency is set to be equal to the excitation frequency before optimizing power into the electrical domain.

This two-stage process, although facilitated an approach to problem solving, rendered incomplete optimization which considered only the electrical damping ratio and left out the influence of the excitation frequency and mechanical damping ratio. Although in a later study by Tang and Zuo [16], a more rigorous optimization analysis that simultaneously considered the electrical damping ratio and normalized excitation frequency was performed, their results were again based on a few examples of mechanical damping ratios, and therefore inadvertently reached the same incorrect conclusion.

Furthermore, all of the above work assumed that the displacement amplitude of base excitation is constant when conducting optimization. However, later in this paper, we will show that the optimization conditions under excitations with constant displacement amplitude and constant acceleration amplitude are actually different. Furthermore, we will show that the long-believed optimization condition is only valid when the acceleration amplitude is constant and an approximation for small mechanical damping ratio, e.g., $<2\%$, when the displacement amplitude is constant.

A similar two-stage process can also be found in several studies of piezoelectric energy harvesting [12,13]. In the work of Guyomar et al. [12], the optimal resistive load is derived after the excitation force and the speed of the mass are considered to be in phase, which is equivalent to matching the natural frequency in prior to matching the electrical impedance, and only an approximation that is good for structures with low viscous damping. Furthermore, in the work of Lefeuvre et al. [13], the same "in-phase" procedure of Guyomar et al. [12] is employed before deriving the optimal resistive load. Lastly, in the work of duToit et al. [11], only impedance matching is considered while frequency matching is left unattended. Later in this paper, we will show that this two-stage, sequential consideration of frequency and impedance matching will lead to erroneous results if the electromechanical coupling is moderate.

Hence, the motivation of this paper is to address these issues that have been repeated and propagated by different research groups, and provide the necessary corrections for the existing and future researchers in energy harvesting. To this end, a two-variable optimization analysis, simultaneously involving the normalized excitation frequency and electrical damping ratio (electromagnetic transduction) or normalized resistive load (piezoelectric transduction), will be performed. Finally, the analysis results will show that the aforementioned two-stage process lead to approximated optimization conditions that are only valid under certain cases of mechanical damping and electromechanical couping. More general optimization conditions will be derived in this paper.

2. Energy harvesting from base excitation

2.1. Harvester with electromagnetic transduction

Fig. 1a shows a SDOF vibration energy harvester with electromagnetic transduction, consisting of a spring-mass-damper system which is subjected to base excitation from ambient vibrations. This model has been widely employed in studying electromagnetic energy harvesting from ambient vibrations [7,14,16]. The equation of motion of this model can be readily derived by Newton's 2nd law as

$$\begin{cases} m\ddot{x} + c_m(\dot{x} - \dot{y}) + k(x - y) + f_e = 0 \\ f_e = \dfrac{k_t k_e(\dot{x} - \dot{y})}{R_H} \end{cases} \tag{1}$$

In Eq. (1), f_e is the force induced by the electromagnetic transducer (electromotive force), R_H is the electrical resistance of the harvester, and k_t and k_e are the motor constant and the electromotive force coefficient (EMF), respectively. Furthermore, c_m is the mechanical damping of the system, k is the stiffness, and m is the seismic mass. Lastly, x and y are the displacement of the seismic mass and base excitation, respectively. Based on Eq. (1), the electromotive force f_e can be thought of as a viscous damping force, which is valid for electromagnetic transducer with resistive load [6]. As result, the equivalent electrical damping due to energy harvesting circuitry can be defined as

$$c_e = \frac{k_t k_e}{R_H} \tag{2}$$