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Limits on the power available to harvest from broadband random excitation

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ABSTRACT

With the large diversity in energy harvesters aiming to extract maximum power from broadband excitations, it is important to know what the maximum power achievable is. This paper derives new upper bounds on the available power for a harvester with general nonlinear stiffness coupled to a nonlinear electrical circuit. White noise base excitations are known to input power proportional to the total oscillating mass of the system and the magnitude of the spectral density of the noise regardless of the details of the oscillating system. This power is split between undesirable mechanical damping and useful electrical dissipation with the form of the stiffness profile and device parameters determining the relative proportions in each dissipation mechanism. An upper bound on electrical power is sought and, provided certain conditions are met, shown to be a simple function of relatively few system parameters and, importantly, independent of the stiffness profile or electrical nonlinearity.

The benefits of knowing the upper limits on power are threefold: to guide optimal harvester design, to assess how close to optimal current devices are and to provide a preliminary estimation of the harvester mass necessary in a given operating environment for a given power requirement.

1. Introduction

As technology develops, the power consumption of electronic devices is decreasing rapidly. Consequently, there is substantial interest in harvesting energy from ambient sources, such as vibration, in order to power small-scale wireless devices where battery replacement or wiring is impractical or unnecessary. The vibrations of an engineering application can be used to excite an electromechanical oscillator that converts the kinetic energy into electrical energy. A number of energy conversion techniques and system designs have been investigated and are comprehensively reviewed in [1–4].

The ability of an energy harvester to extract the maximum power from a given excitation will strongly depend on the characteristics of the excitation. A large proportion of applications will be dominated by harmonic vibrations at one or more fixed or time-varying frequencies and substantial research has been undertaken to develop optimal energy harvesters under these conditions [3,5–7]. However, many applications will vibrate randomly with a broad frequency range often modelled as white [8–19] or other [11,12,20–27] noise.

It is critical in energy harvesting to investigate what the maximum power available from a given excitation is and what type of system can achieve it. For a general force input, an analysis of input energy subtracted by power dissipated under

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various constraints can be made to devise a strategy for maximum power transfer [28–30]. This is effective for deterministic or relatively consistent vibrations, but less capable of handling random excitation. Additionally, unphysical or difficult to realise responses are often found to provide the optimal velocity profile although [30,31] suggest a type of system capable of performing well.

For white noise base acceleration, it has been shown that the power harvested is proportional to the oscillating mass and the noise intensity and independent of the system used to dissipate it. This result has been partially shown or derived for simple or specific systems in a number of ways [10,11,14–16], but the most general and complete proof is that of [8], extended in [9]. Here it is shown that for an arbitrary degree of freedom system with general nonlinearity dependent on displacement and velocity and excited by stationary white noise base acceleration, the power input, P, from the base excitation is

$$P = \frac{\pi S_0 M_{\mathrm{Tot}}}{2} \tag{1}$$

where S_0 is the single-sided spectral density of the excitation and M_{Tot} is the total oscillating mass. This result is independent of the form of the transduction mechanism and when an electrical circuit is present, the power will be split between electrical and mechanical dissipation. A tighter upper bound more specifically for electrical power dissipation is derived in [16] where a typical harvester circuit is coupled to the mechanical oscillator and it is found that a low frequency device increases the power bound.

White noise is an idealisation of realistic noise and is a valid approximation when the bandwidth of the oscillator is significantly narrower than the bandwidth of the noise. It has been shown in [9] that for a wide class of systems, those that exhibit detailed balance, the power calculated using Eq. (1) will be greater than or equal to the power dissipated under non-white excitation where the peak of the spectrum is taken as S_0. The power from white noise excitation can therefore be thought of as an upper bound for these types of systems, which include a single-degree-of-freedom system with a linear dissipation mechanism.

Many studies into optimal systems for harvesting white noise excitations have been undertaken, for example [11–15,17–19], often with a particular focus on the potential of nonlinear systems to improve power transfer. In general, the best system depends on the model of the electrical circuit; if only a dissipative component is used, the power can be simply found from the ratio of mechanical to equivalent electrical damping and is independent of stiffness nonlinearity. However, in reality the electrical system is more complex than containing a simply dissipative component, with piezoelectric and electrostatic circuits containing a capacitor and electromagnetic ones containing an inductor. With these included, it is found that the stiffness potential used will affect the power output. Furthermore, a number of studies, particularly experimental ones, use non-white excitation with a variety of spectra [11,12,20–23,32,25–27] and generally find that the power output depends more strongly on the type of noise and system investigated.

One type of design of great interest in the literature for both harmonic and broadband excitation is the bistable system [12,18,20,21,25,33–37]. It has been concluded both theoretically and experimentally that these systems improve power generation under broadband random excitation when an appropriate electrical circuit is used, although the exact form of the stiffness profile must be tuned to the excitation level in order to achieve inter-well dynamics. This paper agrees with these results and illustrates how it is that these devices approach maximum power when compared to monostable alternatives.

The aim of this paper is to derive an upper bound on the power available to harvest from white noise excitation. Differently from the common approach of optimising a chosen stiffness profile, the bound encompasses all stiffness nonlinearities and as such allows for easy comparison between the diverse range of energy harvesters and illuminates what characteristics in a harvesting system are required to provide maximum power. In what follows, Section 2 derives the power bound, Section 3 uses numerical simulations of a number of popular devices and compares them to the power bound of Section 2 and finally, Section 4 discusses desirable characteristics of optimal harvesters.

2. An upper bound on power harvested

The present analysis is concerned with a single-degree-of-freedom energy harvester as shown in Fig. 1 which consists of a mass, m, that is connected to a vibratory surface via a linear damper of rate b and a nonlinear spring with restoring force $g(x)$, where x represents the displacement of the mass in relation to the vibrating base. An electrical circuit is coupled to the mass consisting of a capacitor of capacitance C and a nonlinear resistor such that the governing equations are

$$m\ddot{x} + b\dot{x} + g(x) + \theta V = -m\ddot{\xi}(t) \tag{2}$$

$$C\dot{V} + \frac{f(V)}{\gamma} = \theta\dot{x} \tag{3}$$

where θ is the electrical coupling coefficient, V is the voltage over the nonlinear resistor and γ is the nonlinear resistance such that $f(V)/\gamma$ is the current flowing through the resistor. $\ddot{\xi}(t)$ represents the white noise base acceleration with autocorrelation function at a time lag τ of $\pi S_0 \delta(\tau)$ where S_0 is the single-sided spectral density and $\delta(\tau)$ is the delta function.