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Nonlinear vibration analysis of the resonant column test of granular materials

Song-Hun Chong [a], Jin-Yeon Kim [b,*]

[a] School of Civil and Environmental Engineering, Georgia Institute of Technology, 790 Atlantic Drive, Atlanta, GA 30332, USA
[b] GWW School of Mechanical Engineering and School of Civil and Environmental Engineering, Georgia Institute of Technology, 790 Atlantic Drive, Atlanta, GA 30332, USA

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ABSTRACT

This paper theoretically analyzes the nonlinear torsional vibration of a granular material specimen in the resonant column test (RCT). The RCT is a material testing method that has been well established in geotechnical engineering. The test provides some important dynamic properties of granular materials including the shear wave speed, effective dynamic shear modulus and damping capacity. Both the elastic modulus and damping capacity show a strong dependence on the amplitude of excitation, which is the essential behavior of materials with hysteretic nonlinearity. The analysis, starting with a constitutive relationship that describes the hysteretic nonlinear behavior of granular materials, is performed specifically on the forced torsional vibration of a circular bar with a fixed and inertia-loaded boundary condition as a model for the specimen in the RCT. Self-interaction of a single resonance mode is assumed in the perturbation analysis to obtain an approximate solution for near-resonance response. The theoretical results predict linear dependences of the resonance frequency and damping ratio on the shear strain; these are in agreement with the existing experimental observations. Notable is that the present model describes the hysteretic nonlinear behaviors with a single parameter, i.e., the hysteresis nonlinearity parameter in the constitutive relationship. The derived equations are applied to a jointed gneiss specimen where how the hysteresis nonlinearity parameter is determined from RCT data is demonstrated.

1. Introduction

In geotechnical engineering, the RCT has been used for last several decades to characterize dynamic properties of granular materials including soils, jointed rocks, etc [1–4]. The principle of the RCT is to induce torsional vibration of a cylindrical specimen in a small to intermediate range of shear strain and measure the frequency response curves around the specimen's fundamental resonance frequency. The shear modulus and damping ratio of the specimen are calculated from the resonance frequency and the bandwidth of the response, respectively. The torque-strain curve for the harmonic excitation exhibits hysteresis [5]; the strength of the hysteresis increases with excitation amplitude. Bolton and Wilson [5] proposed a model for soil hysteresis behavior based on the distributed element model of Iwan [6] in which each Jenkin element [7] follows an ideal elastoplastic hysteresis curve but with a different hysteresis strength. They attempted to fit the

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[*] Corresponding author.
E-mail address: jykim@gatech.edu (J.-Y. Kim).

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model to experimental data. The hysteretic behavior of particles results in the strain-dependent (or excitation amplitude dependent) resonance frequency and damping ratio. These nonlinear phenomena observed in granular materials are not fully understood and an accurate theoretical analysis has not been available to the geotechnical engineering community. More specifically, without an explicit form of the stress-strain relationship for such hysteretic nonlinear behaviors of granular materials, an analysis of the vibration of RCT specimen, which will provide an exact relationship between the dynamic shear modulus and the shear strain, cannot be performed. For this reason, the characteristic equation for infinitesimal strains (linear vibration) has been adopted to calculate the effective dynamic shear modulus. The use of the characteristic equation for linear vibration obviously will lead to incorrect results. Most of investigations have rather focused on fitting the effective dynamic shear modulus using an empirical function such as the Hardin and Drnecivh model [2] and the Ramberg and Osgood [8] model. Currently, an analytical treatment of the vibration of the specimens under RCT is not existing.

Over the last two decades, the geophysics community has made a significant progress in physical understanding of the nonlinear elastic behavior of granular materials [9–11]. Hysteresis and end-point memory in the stress-strain curve, higher harmonic generation in propagating waves, resonance frequency shift and nonlinear damping in standing waves, and slow dynamics are some of the nonlinear phenomena observed in these materials. The mechanisms that cause these nonlinear behaviors are at mesoscopic scales and include friction, contact, adhesion, rupture of bond between grains, microfracture, and so on [10,12]. Phenomenological models based on the Preisach-Mayergoyz space representation are proposed to describe the resulting hysteretic nonlinear behavior in the stress-strain curve [13,14].

This paper analyzes for the first time the nonlinear torsional vibration of the specimen in the RCT from the viewpoint of hysteretic nonlinear elasticity. The analysis employs the recently established constitutive model for granular materials [15]. Then, a perturbation technique is applied based on a single mode self-interaction approximation. The analysis technique is formally similar to that of Van Den Abeele [16] for the nonlinear longitudinal vibration. The objective is to provide the formulas for the resonance frequency and damping ratio for the specific purpose of interpreting the data from the RCT. Using these formulas and experimental data, one can determine the hysteresis nonlinearity parameter. With the hysteresis nonlinearity parameter determined in this way, one has a complete analytical form of the constitutive relationship for the granular material under consideration. Other material properties of geotechnical interest such as the back bone curve and effective dynamic shear modulus can be obtained from the constitutive relationship. The present paper lays down the first step toward a completely new procedure of interpreting the data from the RCT. As an example of application, the theoretical results are applied to the experimental data for a jointed gneiss specimen.

2. The resonant column test

A typical experimental setup for the RCT is shown in Fig. 1 [4]. The torsional vibration of the hollow cylindrical specimen is induced by external twisting moment generated from an electromagnetic torsional shaker. The moment is directly exerted on the drive plate that clamps the specimen's top. Two accelerometers that are attached at two diagonally opposite locations on the drive plate measure the tangential acceleration signals. Then, the tangential acceleration is converted to torsional strain. The electromagnetic shaker excites the specimen-drive plate system in a frequency band around the specimen's lowest resonance frequency. The specimen has a hole through which a dead load is hanging to apply a static stress on the specimen. By measuring the torsional responses for varying static load, one can evaluate the dependency of the resonance frequency and damping capacity of the specimen on the static compressive load. Knowledge of the dependency is of great importance, for example, in the near-surface geophysical characterization. The specimen bottom is fixed to the support frame the entire structure of which is mounted on rubber dampers to suppress the vibration of the structure (Fig. 1). In the RCT, various types of specimens are tested including homogeneous specimens (rock, sand, clay) and inhomogeneous jointed rocks (rock-gouge system), etc. Even for the case of jointed rocks, the whole specimen can be viewed as an effective homogeneous material since the specimen is prepared such that the wavelength at the lowest resonance is sufficiently larger than the single rock joint unit. Therefore, the vibrating system can be modeled as a homogeneous hollow cylinder fixed on one end and mass-loaded on the other end with an external twisting moment applied at the loaded mass.

The conventional RCT apparatus designed for soil testing are not capable of testing rocks and/or jointed rocks because of the higher stiffness and larger size of rock specimens. The insufficient torsional strain of the conventional apparatus is the compelling limitation especially when the strain-dependent behavior of rock specimens is to be investigated. The rock mass dynamic test apparatus developed by Chong et al. [4] is a universal RCT apparatus that can generate a wide range of torsional strain (up to 10^{-4}) and can accommodate large-sized specimens. More information on the rock mass dynamic test apparatus can be found in [4].

Chong et al. [4] performed RCTs on a gneiss specimen. A cored NX-size gneiss specimen was sliced into discs with thickness 25.4 mm using a diamond saw. The discs were then polished to have their surfaces smooth and flat. A hole of diameter 25.4 mm was drilled at the center of the discs. The basic parameters of the gneiss specimen are shown in Table 1. Twelve discs are stacked up to form a hollow cylindrical specimen and are held together by a static, axial, compressive load. Three different levels of axial load, 394 kPa, 580 kPa, and 766 kPa, corresponding to weights of 10.65 kgf, 10.60 kgf, and 20.66 kgf, are applied to investigate the effects of normal stress on the shear behavior of gneiss under a large strain. Typical frequency responses of a gneiss specimen for varying strain amplitude are shown in Fig. 2 [4]. The resonance frequency and

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