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Inner detached frequency response curves: an experimental study

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ABSTRACT

Certain nonlinear vibrating systems have frequency response curves (FRCs), in which isolated detached curves exist inside the main continuous FRC. The behavior of these systems has hitherto been studied analytically and numerically, but to the authors' knowledge, there is no record of an inner detached FRC being detected experimentally. These curves may be hidden by numerical or experimental analysis, particularly when a system is subject to swept or stepped-sine excitation. Their existence may thus lead to unexpected dramatic changes in the amplitude of the system response. This paper presents an experimental study that involves the design, construction and testing of a specific system that has an isolated detached FRC inside the main continuous FRC. The experimental design of the test rig is supported by multibody dynamic simulations, and in the experimental tests the existence of a detached FRC was verified.

1. Introduction

One interesting feature of nonlinear oscillating systems is the multi-valuedness of their frequency response curves (FRCs) at some frequencies of excitation [1]. Specifically, in the case of harmonic excitation, and provided that the system response is predominantly harmonic at the excitation frequency, multiple solutions may appear in the steady-state amplitude response at a single frequency. Depending on the system configuration and on the values of the system parameters, closed detached resonance curves can appear either outside or inside the main continuous FRC.

Outer detached resonance curves have been predicted by several authors. For example, Starosvetsky and Gendelman [2] reported their theoretical prediction and numerical confirmation when analyzing the performance of a nonlinear vibration absorber attached to a linear host structure; they also predicted similar features when analyzing a three degrees-of-freedom (DOF) nonlinear oscillating system [3]. Alexander and Schilder [4] discovered a family of outer detached curves when analyzing the performance of a nonlinear tuned mass damper containing cubic stiffness nonlinearity. Outer detached curves have also been identified by Kerschen and co-workers [5,6] for the forced response of different nonlinear structures. More recently, isolated response regions outside the main continuous FRC have been shown to be also caused by internal resonances in simulations and experimental tests [7].

Inner detached resonance curves appear to have been first theoretically predicted and investigated by the authors of this paper and a co-worker [8] – they were validated numerically and named *bubbles* after their shape. By deriving approximate equations for the FRC of a harmonically excited system consisting of a coupled nonlinear and linear oscillator, the effect of

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system parameters on their appearance was subsequently investigated [9,10]. The main restriction in their analysis, however, was a very low mass ratio between the nonlinear oscillator and the linear oscillator. This limitation meant that it was very difficult to design a test rig for experimental validation, as it became evident that isolated curves can only occur for a very narrow range of some of the system parameters [9].

Very recently, a comprehensive investigation on the detection of inner detached resonance curves has been reported by Gatti [11]. In this work, approximate analytical expressions for the FRC of a 2-DOF nonlinear oscillator were derived without any limitations on the mass ratio. Building on the previous analytical and numerical studies, this paper describes the design of a specific experimental rig, and subsequent tests to show conclusive evidence of an inner detached FRC in a 2-DOF system consisting of a coupled linear and nonlinear oscillator excited by a harmonic force. To design the test rig, a multibody model of the system was used to investigate the effect of un-modeled parameters in the analytical model, such as gravity, detailed geometry and rotational inertia, which could affect the dynamic behavior in a practical system.

The paper is organized as follows. Following this Introduction, an overview of the phenomenon of detached resonance curves is presented. Section 3 then describes the experimental design and Section 4 is devoted to a description of the experimental tests and results. The paper is concluded in Section 5. There is also an Appendix, which contains a resume of the pertinent results from reference [11].

2. Systems that have a detached FRC: an overview

Probably the simplest oscillating system where a detached FRC is theoretically predicted to occur, is a 2-DOF system, consisting of a linear spring-mass-damper system excited by a harmonic force, with an attached nonlinear oscillator having a linear plus cubic stiffness nonlinearity of the hardening type [8]. A schematic of such a system is shown in Fig. 1, whose equations of motion are given by

$$\begin{matrix} m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s + c_1\dot{z} + k_1 z + k_3 z^3 = F\cos(\omega t) \\ m\ddot{x}_s - m\ddot{z} - c_1\dot{z} - k_1 z - k_3 z^3 = 0 \end{matrix}, \qquad (1a,b)$$

where m_s, k_s and c_s are the mass, stiffness and damping coefficient of the linear system, respectively; F and ω are the force amplitude and angular frequency of excitation, respectively; m and c_1 are the mass and damping coefficient of the attached nonlinear system, respectively; k_1 and k_3 are the linear and cubic stiffness coefficients, respectively; x_s and x are the displacements of the masses, and $z = x_s - x$ is the relative displacement; the overdots denote differentiation respect to time t.

In previous work [11], for small values of the damping ratios $\zeta_s = c_s/2m_s\omega_s$ and $\zeta = c_1/2m\omega_s$ (where $\omega_s = \sqrt{k_s/m_s}$), say less than about a few percent, it was found that two key non-dimensional parameters influence the condition that is necessary for the existence of an inner detached FRC. They are the mass ratio given by $\mu = m/m_s$, and the ratio between the natural frequency of the nonlinear attachment with $k_3 = 0$, to the natural frequency of the linear system alone, given by $\Omega_0 = \sqrt{k_1/(\mu k_s)}$. The necessary condition for the appearance (or not) of an inner detached FRC for an attachment with hardening stiffness nonlinearity is shown as a function of these two variables in Fig. 2, which is adapted from [11] – the meaning of the markers in this figure is given later. An inner detached FRC can potentially exist for the range of parameters corresponding to the white region of Fig. 2, while it will not exist for the combination of parameters corresponding to the shaded region. Note that an inner detached FRC may only exist when the natural frequency of the attachment (with $k_3 = 0$) is lower than the natural frequency of the linear system, and for relatively low values of the mass ratio. This occurs because

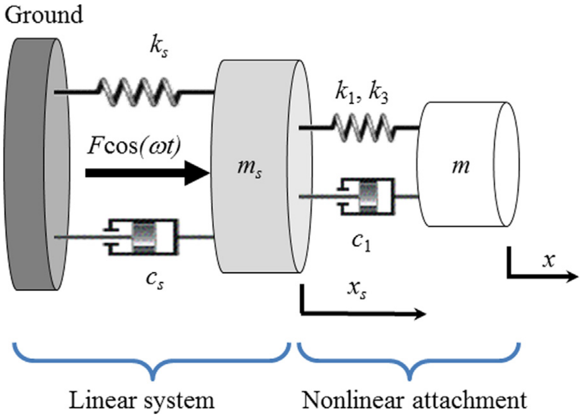

Fig. 1. Model of the 2-DOF nonlinear system under consideration.