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Damage localization from projections of free vibration signals

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ABSTRACT

An approach for damage localization that operates with free vibration signals in the damaged state is presented. The scheme does not directly point to the damage location but, given a postulated distribution, computes a metric that (for ideal conditions) approaches infinity when the distribution is correct. The approach does not use identification, operates without constraints between the number of sensors and the number of model degrees of freedom and differs from model updating in that only the damage distribution (and not the severity) enters the formulation. Analytical and experimental results are included.

1. Introduction

This paper presents a scheme for damage localization that operates without identification and which applies whenever it is possible to extract free-vibration signals from the damaged state. The approach applies, therefore, in structures that can be excited using impact, or when free vibration can be synthesized with sufficient accuracy from forced response, typically using the Random Decrement technique [1–6]. The method, designated as the Free Vibration Projection Damage Localization (FvPDL) approach, is derived under the common assumptions of finite dimensionality and linear behavior and operates within theory when the number of sensors, m, is no less than the rank of the change in the transfer matrix resulting from damage. In the common case where damage is idealized as changes in the stiffness of the model that replaces the real structure the constraint reduces to $m > Rank(\mathbf{K}_D - K_R)$; where K stands for stiffness matrix and the subscripts D and R refer to the damaged and the reference states, respectively. This constraint is the same that applies in SDLV and SDDLV [7, 8], as well as in the recently introduced S3DL scheme [9].

It's opportune to note that FvPDL does not "point to the damage" but rather provides a metric that is correlated with the "likelihood" that a postulated damage pattern is the correct one; where likelihood has been placed in quotes to indicate that it's been used colloquially and not in the precise sense that it has in probability theory. The metric is a measure of the parallelism between a particular projection of the Fourier transform of the damaged state free vibration and a matrix whose span depends on the postulated damage distribution. The metric is invariant with respect to post-multiplication of the damage distribution by and arbitrary full rank matrix and it thus follows that (for ideal conditions) results are independent of damage severity. The approach is devoid of heuristics and thus guaranteed to succeed when the assumptions used in the derivation are met. The relevant question is, therefore, whether it can operate with sufficient robustness against the

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limitations imposed by model error and noise in the measurements. To gain some insight into this matter care is taken in the numerical section to incorporate reasonable levels of model error and measurement noise and results from an experimental examination are also included. In its present form the approach is practically restricted to situations where the number of possible damage distributions is manageable.

While there are a number of studies on damage localization were measurements are pre-processed into free decay, these studies do not offer a theoretically grounded connection between the free vibration signals and the damage location but either: 1) used the free decay to perform modal identification, followed by attempts to locate the damage from mode shape changes [10–12], or 2) used heuristics to map the changes in the free decay signals, typically synthesized using the Random Decrement approach, to the damage [13,14]. FvPDL, in contrast, does not use system identification or pattern recognition and, as noted, offers a theoretical link between free vibration signals in the damaged state and the spatial distribution of the damage. Another point of contrast with the techniques of ref. [10–14], is the fact that FvPDL is not restricted to specific structural topologies, e.g., beams of frames but applies to any system that has been discretized and modeled. The paper is organized as follows: derivation of the expressions that constitute the approach is presented in Section 2. Sections 3 and 4 describe the approach from an implementation perspective and show results of some numerical experiments. Section 5 shows experimental results obtained when "the damage" is a mass perturbation in an aluminum plate and the paper closes in Section 6 with a brief critical review. An Appendix A containing the expressions needed to implement the approach using a truncated set of modes, as is computationally necessary when the model is large, is also included.

2. Damage localization from free decay response

Accepting the typical assumptions of linearity and finite dimensionality and assuming that damage can be treated as a change in the stiffness matrix ΔK the response to an arbitrary initial condition in the damaged state can be written as

$$M\,\ddot{x}_d(t) + C\,\dot{x}_d(t) + (K - \Delta K)x_d(t) = 0 \quad x_d(0) = x_0,\ \dot{x}_d(0) = \dot{x}_0 \tag{1}$$

From Eq. (1) one can write

$$M\ddot{x}_h(t) + C\dot{x}_h(t) + Kx_h(t) = 0 \quad x_h(0) = x_0,\ \dot{x}_h(0) = \dot{x}_0 \tag{2}$$

and

$$M\ddot{x}_p(t) + C\dot{x}_p(t) + Kx_p(t) = \Delta K x_d(t) \quad x_p(0) = 0,\ \dot{x}_p(0) = 0 \tag{3}$$

where it is clear that we've taken

$$x_d(t) = x_h(t) + x_p(t) \tag{4}$$

It's opportune to note that although we make reference to the initial state,x_0, and to the change in stiffness due to damage, ΔK, these quantities are not used in the final implementation. Solving Eqs. (2 and 3) in Laplace and adding the superscript o to indicate measured coordinates one has

$$x_h^o(s) = R(s)\begin{Bmatrix} x_0 \\ \dot{x}_0 \end{Bmatrix} \tag{5}$$

and

$$x_p{}^o(s) = T(s)\,\Delta K x_d(s) \tag{6}$$

where

$$R(s) = C_u \cdot G(s) \cdot L(s) \tag{7}$$

with

$$L(s) = \left[(M \cdot s + C) \quad M \right] \tag{8}$$

and

$$T(s) = C_u G(s) \tag{9}$$

where $C_u \in R^{m \times n}$ is a matrix that selects the rows that corresponds to sensor locations and $\mathbf{G}(s) = (Ms^2 + Cs + K)^{-1}$ is the model force to displacement transfer matrix. Since the initial state in Eq. (5) is not a function of the Laplace variable the expression can be evaluated at k-values of s and stacked. Combining the stacked equation with the definition introduced in Eq. (4) one has

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