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A double expansion method for the frequency response of finite-length beams with periodic parameters

Z.G. Ying [a,*], Y.Q. Ni [b]

[a] Department of Mechanics, School of Aeronautics and Astronautics, Zhejiang University, Hangzhou 310027, PR China
[b] Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China

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ABSTRACT

A double expansion method for the frequency response of finite-length beams with periodic distribution parameters is proposed. The vibration response of the beam with spatial periodic parameters under harmonic excitations is studied. The frequency response of the periodic beam is the function of parametric period and then can be expressed by the series with the product of periodic and non-periodic functions. The procedure of the double expansion method includes the following two main steps: first, the frequency response function and periodic parameters are expanded by using identical periodic functions based on the extension of the Floquet-Bloch theorem, and the period-parametric differential equation for the frequency response is converted into a series of linear differential equations with constant coefficients; second, the solutions to the linear differential equations are expanded by using modal functions which satisfy the boundary conditions, and the linear differential equations are converted into algebraic equations according to the Galerkin method. The expansion coefficients are obtained by solving the algebraic equations and then the frequency response function is finally determined. The proposed double expansion method can uncouple the effects of the periodic expansion and modal expansion so that the expansion terms are determined respectively. The modal number considered in the second expansion can be reduced remarkably in comparison with the direct expansion method. The proposed double expansion method can be extended and applied to the other structures with periodic distribution parameters for dynamics analysis. Numerical results on the frequency response of the finite-length periodic beam with various parametric wave numbers and wave amplitude ratios are given to illustrate the effective application of the proposed method and the new frequency response characteristics, including the parameter-excited modal resonance, doubling-peak frequency response and remarkable reduction of the maximum frequency response for certain parametric wave number and wave amplitude. The results have the potential application to structural vibration control.

1. Introduction

Structural dynamic characteristics and frequency response are significant for structural dynamic design including vibration control and optimization. The engineering structures with spatial periodic distribution parameters such as

* Corresponding author.
E-mail address: yingzg@zju.edu.cn (Z.G. Ying).

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periodic geometric parameters, periodic physical parameters and periodic constraints have been considered largely [1–43]. The studies include characteristic frequencies, modal localization and buckling [2–13], quasi-periodic distribution parameter effects [14–18] and application [32–38]. The parametric periodicity has a great influence on the structural dynamic response due to the coupling of structural response and periodic distribution parameters. The periodic structural systems with space-varying parameters have the dynamics different from the linear constant-coefficient systems. Several analysis methods for the periodic structures have been proposed, including the transfer matrix method and spatial harmonic expansion method (or Bloch wave expansion method) [1,22,24–27]. In many studies, the spatial periodic dynamics were considered and the periodicity conditions were used. The structural dynamic response usually does not have the same period as the parameters.

Engineering structures generally have finite sizes and boundary constraints. The boundary conditions have a remarkable effect on the dynamic response of the finite-size structures with periodic distribution parameters [39–42]. Several analysis methods such as the spatial harmonic expansion method (or Bloch wave expansion method), transfer matrix method and finite element method have been applied to the finite-size periodic structures [1,28,39,41]. However, as the period-parametric wave number increases, the smaller element sizes of the spatial discretization or the more terms of the modal superposition expression need to be considered in the analysis. The finite element method has the capability limited by the high parametric wave number and high frequency response. For the direct spatial harmonic expansion method, the analysis accuracy is limited by the unconsidered high modes, because the effects of the periodic parameters and high-order modes on the dynamic response are coupled each other. For example, the high parametric wave number combined with high-order modes can affect low-order mode components, so that much more modes or larger numbers of expansion terms are required even for low frequency response. Therefore, the analysis method for the dynamic response of finite-size structures with periodic distribution parameters needs to be studied further and extended.

An analysis method for the dynamic response of the periodic structures developed needs to uncouple the effects of the periodic parameters and modal expansion on the response. The stability of structural systems with temporal periodic parameters has been studied based on the Galerkin method and Floquet theorem [44,45], and the analysis procedure can be recomposed for the structural systems with spatial periodic parameters. Based on the extension of the Floquet-Bloch theorem, the structural response and periodic parameters can be firstly expanded by using identical periodic functions. The structural differential equation with spatial periodic parameters can be converted into a series of linear differential equations with constant coefficients, which depend on only the periodic parameter expansion. Then based on the Galerkin method, the solutions to the linear differential equations can be expanded by using modal functions. The linear differential equations can be converted into respectively a series of vibration equations and the vibration responses can be finally determined, which depend on only the modal expansion. The above double expansion method can uncouple the effects of the periodic expansion and modal expansion and make the expansion terms determined respectively, which is illustrated by the periodic beam in the present research.

In this paper, a new double expansion method for the frequency response of periodic structures is proposed, and the vibration response of a finite-length beam with spatial periodic parameters under harmonic excitations is studied. Firstly, the differential equation of motion of the finite-length periodic beam and the differential equation for the frequency response are given. The Floquet-Bloch theorem is extended for the frequency response of periodic structures. It is demonstrated that the frequency response function can be expressed by the product of periodic and non-periodic function vectors, which period is equal to the parametric period. Secondly, the frequency response function and periodic parameters are expanded by using identical periodic functions such as harmonic functions. The number of expansion terms is determined based on the periodic parameters. The parameter-varying differential equation for the frequency response is converted into a series of linear differential equations with constant coefficients. Thirdly, the solutions to the linear differential equations are expanded further by using modal functions which satisfy the boundary conditions. The modal number is determined based on the vibration frequency. The linear differential equations are converted into algebraic equations according to the Galerkin method. Fourthly, the expansion coefficients are obtained by solving the algebraic equations, and then the frequency response function is determined as a double series. The proposed double expansion method is illustrated by a simply supported beam with periodic parameters. Finally, numerical results on the frequency response of the periodic beam with various parametric wave numbers and wave amplitude ratios are given to show the effective application of the proposed method and the frequency response characteristics. The proposed double expansion method uncouples the effects of the periodic expansion and modal expansion, and converts the period-parametric differential equation into linear differential equations with constant coefficients. New frequency response characteristics such as the parameter-excited modal resonance, doubling-peak frequency response and remarkable reduction of the maximum frequency response for certain parametric wave number and wave amplitude are revealed.

2. Vibration equation of periodic beam and its expansion principle

Consider a horizontal elastic beam with longitudinal periodic parameters as shown in Fig. 1. The vertical vibration equation of the beam in terms of the Euler-Bernoulli model can be expressed as [46].

$$\rho A\ddot{w} + c_d\dot{w} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = f(x, t) \tag{1}$$