Journal of Sound and Vibration 388 (2017) 1–19

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Boundary control design for extensible marine risers in three dimensional space

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ARTICLE INFO

Article history:
Received 16 January 2016
Received in revised form
21 August 2016
Accepted 9 October 2016
Handling Editor: L.G. Tham
Available online 5 November 2016

Keywords:
Marine riser
Boundary control
Evolution system
Practical exponential stability
Hilbert space

ABSTRACT

A design of boundary controllers is proposed for (practical) exponential stabilization of extensible marine risers in three-dimensional (3D) space under sea loads. The design removes flaws in existing works. Two Lyapunov-type theorems are developed for study of existence and uniqueness, and stability of nonlinear evolution systems in Hilbert space. These theorems have their potential use in control design and stability analysis for flexible systems including marine risers.

1. Introduction

In addition to transporting petroleum products from the sea-bed to a surface ship/rig, marine risers, which are slender beams/pipes, are used in various offshore installations [1,2]. To reduce the riser vibration induced by sea loads, distributed control and/or boundary control can be used. The distributed control requires an infinite number of actuators implemented along the riser. Although the distributed control can lead in desired results (such as state-constraints and strong robustness to disturbances via the use of various control design methods using Barrier Lyapunov Functions [3,4] and backstepping technique [5]), it is impractical to be implemented because it requires an infinite number of actuators and introduces significant drag to the riser. On the other hand, the boundary control only requires measurements and actuators implemented at the top-end of the riser. Thus, the boundary control does not introduce drag to the riser and is often used. Boundary control of both inextensible and extensible marine risers has been extensively considered, see [6–18] and other related works on boundary control of beams [19], strings [20–22], strips [23,24]. However, the control design is only matured for 3D inextensible risers [9] and one/two-dimensional (transverse motions) risers in [8,14]. It is noted that in [10] although extensive risers were considered but strain energy is ignored. Boundary control of extensible marine risers was initially considered in [11,15]. However, the problem of choosing the design constants to make the time derivative of Lyapunov function candidates negative definite was left open in both [11,15]. Moreover, the control design in [6,7,11–13,17,18,25] has flaws, which are either (1) assuming stability to prove stability or (2) wrong use of Sobolev embedding as discussed in what follows.

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http://dx.doi.org/10.1016/j.jsv.2016.10.011
0022-460X/

1.1. Assuming stability to prove stability problem

In [7, page 196], the authors made an assumption that the riser/beam is not exposed to compression at any point and at any time so that $\frac{EA}{2}\eta_x^2 + EA\mu_x$ with E, A, η_x, μ_x being Young's modulus, cross-section area, slopes of transverse and axial motions, is always nonnegative prior to the control design. It is noted that the term $\frac{EA}{2}\eta_x^2 + EA\mu_x$ can be nonnegative when the riser is not deforming/moving under the pre-tension. However, the riser is deforming during the operation. In certain segments of the riser, η_x^2 can be very small while μ_x can be negative. This occurs when some segments of the riser are exposed to compression. The above discussion leads to a conclusion that the nonnegative assumption on $\frac{EA}{2}\eta_x^2 + EA\mu_x$ can violate at a certain number of points along the riser during operation. The nonnegative assumption on $\frac{EA}{2}\eta_x^2 + EA\mu_x$ is then used to prove negative definite of the time derivative of the Lyapunov functional candidate $\dot{V}(t)$ on page 198 in [7], right column, i.e., the term $-\frac{1}{2}\gamma\int_0^L (P(x,t) - P_0)\eta_x^2 dx = -\frac{1}{2}\gamma\int_0^L \left(\frac{EA}{2}\eta_x^2 + EA\mu_x\right)\eta_x^2 dx$, where $P(x,t) = P_0 + \frac{EA}{2}\eta_x^2 + EA\mu_x$ as defined in (2) in [7], is non-positive due to the nonnegative assumption on $\frac{EA}{2}\eta_x^2 + EA\mu_x$. The above problem means that the authors in [7] assumed negative definite of $\dot{V}(t)$ to prove stability of the closed-loop system.

1.2. Sobolev embedding problem

In [6] on page 125, the inequality $2[v'(x,t)]^2 \le [w'(x,t)]^2$, where $v'(x,t)$ and $w'(x,t)$ are slopes of axial and transverse motions, was used to prove the bounds of the Lyapunov function. The authors of [6] mentioned that they took the above inequality from [26]. However, [26] does not contain this inequality. Actually, the above inequality is wrong as it is clear from the case that the riser is only axially deformed. The Sobolev embedding [27] does not apply to two independent functions $v(x,t)$ and $w(x,t)$. The Sobolev embedding was also wrongly used in [6, page 123, 11, page 1083, 12, pages 725, 726, 13, page 949, 17, pages 5804–5806, 18, page 5025, 25, pages 764, 767], i.e., the correct embedding is $W^{4,2}([0,L]) \to W^{3,2}([0,L]) \to W^{2,2}([0,L])$ not vice versa, where $W^{m,p}([0,L])$ is the Sobolev space of order m and degree p. Specifically, the above works [6,11–13,17,18,25] suffer from the flaw that boundedness of the energy, is a functional of L^2-norm of the first-order spatial derivative of the axial displacement and the second-order spatial derivative of the transverse displacement cannot guarantee boundedness of L^2-norm of higher-order spatial derivatives of these displacements. For example, it is clear from the Sobolev embedding or Poincare's inequality that in [6, page 123] boundedness of $\int_0^L [w'(x,t)]^2 dx$ and $\int_0^L [w''(x,t)]^2 dx$ (in the energy functional) cannot guarantee boundedness of $\int_0^L [w'''(x,t)]^2 dx$ or $|w'''(x,t)|$. The same problem is occurred in [28, page 165].

The above discussion acknowledges contributions of the authors of the works even containing flaws and motivates the writing of this paper on a new design of boundary controllers, which corrects all the aforementioned flaws for (practical) exponential stabilization of 3D marine risers under sea loads. The main contributions of this paper include three folds. First, a mathematical model describing motion of extensible marine risers in 3D under sea loads is derived in an appropriate form for boundary control design. In comparison with the models used in existing works (e.g., [6–12,14,16,18]), properties of the model are derived, see Remark 2.1. Second, two Lyapunov-type theorems are developed to study well-posedness and global (practical) exponential stability of a class of evolution systems in Hilbert space. These theorems just require: (1) continuity and local monotonicity conditions on the system functions; and (2) "usual" conditions on the Lyapunov function and its generator (time derivative). Since the usual conditions are of a form similar to those for lumped-parameter systems in Euclidean space [29], these two Lyapunov-type theorems find a potential application to other distributed-parameter systems (after transformed to an evolution system in Hilbert space) without requirement of carrying out a tedious calculation using the Faedo-Galerkin approximation method as in [8,9,30]. Third, boundary controllers are designed to achieve global well-posedness and (practical) exponential stability of the marine risers based on the proposed Lyapunov-type theorems. This design corrects all the aforementioned flaws in existing works.

2. Problem formulation

2.1. Mathematical model of marine risers in 3D

The riser's configuration is shown in Fig. 1. The boundary configuration in Fig. 1A is used when three boundary control forces provided by a 3D guide tube mechanism are available while the one in Fig. 1B can be implemented on an active heave compensation system [31,32]. Let (u, v, w) denote the displacements of the riser along the OX-, OY-, and OZ-axis from the point N_0 of the reference riser center line to the point N of the deformed riser center line at time t. To derive the equations of motion of the riser, we use the extended Hamiltonian principle [33]: