Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Coefficient of restitution in fractional viscoelastic compliant impacts using fractional Chebyshev collocation

Arman Dabiri *, Eric A. Butcher, Morad Nazari

Department of Aerospace and Mechanical Engineering, University of Arizona, United States

A R T I C L E I N F O

Article history:
Received 27 June 2016
Received in revised form
26 August 2016
Accepted 10 October 2016
Handling Editor: A.V. Metrikine

Keywords:
Impact
Compliant contact model
Continuous contact force model
Fractional order system
Materials with memory
Coefficient of restitution
Kelvin–Voigt model
Maxwell model
Standard linear solid model
Fractional Chebyshev collocation method
Grunwald–Letnikov
Riemann–Liouville
Caputo

A B S T R A C T

Compliant impacts can be modeled using linear viscoelastic constitutive models. While such impact models for realistic viscoelastic materials using integer order derivatives of force and displacement usually require a large number of parameters, compliant impact models obtained using fractional calculus, however, can be advantageous since such models use fewer parameters and successfully capture the hereditary property. In this paper, we introduce the fractional Chebyshev collocation (FCC) method as an approximation tool for numerical simulation of several linear fractional viscoelastic compliant impact models in which the overall coefficient of restitution for the impact is studied as a function of the fractional model parameters for the first time. Other relevant impact characteristics such as hysteresis curves, impact force gradient, penetration and separation depths are also studied.

Published by Elsevier Ltd.

1. Introduction

Impact is defined as the behavior of two or more objects in a collision. In an impact problem, a mathematical model describing the process of impact is obtained [1–3]. Impact problems have been used widely in many areas such as automobile industries, aerospace industries, sport equipment industries, biomedical industries, etc. [4–6]. Various impact problems can be modeled via linear or nonlinear force–displacement relations during the impact interval. Linear viscoelastic compliant impact models must be extended to high orders in order to realistically model certain characteristics of the force–displacement relation which may be possible to model with lower order nonlinear models.

In many models, impact occurs in one dimension and the system hereditary property is approximated by linear viscoelastic constitutive models [1,2,7]. According to the linear theory, a viscoelastic model can be considered as a combination of linear springs and dampers in series and parallel. The Kelvin–Voigt (K–V) viscoelastic model represented by a parallel linear spring damper element is the most commonly used model for impact in one dimension since the impact dynamics are

* Corresponding author.
E-mail address: armandabiri@email.arizona.edu (A. Dabiri).

http://dx.doi.org/10.1016/j.jsv.2016.10.013
0022-460X/Published by Elsevier Ltd.

described by a single-degree-of-freedom vibration model [8]. The K–V model can be used to account for the energy dissipation in one-dimensional compliant impact problems. However, using this simple model for realistic compliant impacts can result in a loss of accuracy since it does not capture certain characteristics of realistic impacts. Moreover, the main drawback of using the linear K–V model is its unrealistic hysteresis diagram for the impact force–displacement curve. Specifically, the discontinuous force profiles at the initial contact and release as well as a nonphysical tensile force applied during the end of the restitution phase are the chief disadvantages of the linear K–V model. Therefore, various higher order linear and nonlinear models which also satisfy the expected boundary conditions have been proposed in the literature [8–10]. For instance, the Hertzian contact stress theory is a more suitable method to take the geometric nonlinearity into account. Although the Hertzian impact model by itself does not include the energy dissipation, it has been modified in the literature to take energy dissipation into account in order to eliminate this force discontinuity [9]. However, this model requires the use of many parameters for a realistic response and does not retain the advantages of using a linear viscoelastic impact model.

Fractional differentiation and integration operators are extended versions of the integer order differentiation and integration operators [11]. Because of this non-local property of fractional operators, it has been shown that fractional calculus provides convenient mathematical tools to model materials with memory or hereditary property [12–14]. For instance, as compared to the integer order linear viscoelastic models with a large number of parameters, fractional viscoelastic models with a small number of parameters can be successfully fitted to the experimental data of viscoelastic materials [15–17]. Consequently, fractional-order differential equations have been used to describe different physical processes with hereditary nature due to their non-local property in applications such as diffusion, biomechanics, viscoelastic materials, time-delayed systems, etc. [18–27].

In this study, it is shown that the use of linear fractional-order impact models allows for the inclusion of realistic characteristics of impacts which traditionally have been accommodated using either nonlinear or high order linear viscoelastic models. Therefore, it is possible to study one-dimensional impact problems in the framework of fractional-order linear viscoelasticity. For this purpose, certain representative impact models are formulated based on the basic fractional viscoelastic models: the fractional K–V model, fractional Maxwell model, fractional K–V standard linear solid (K-SLS) model, and fractional Maxwell standard linear solid (M-SLS) model. A numerical discretization method known as the fractional Chebyshev collocation (FCC) method is used as a numerical approximation tool for the simulation of these impact models. The FCC method has advantages compared with other integration methods for fractional differential equations: (a) it gives a minimal state space representation for commensurate or incommensurate fractional-order models, (b) it obtains the solution with spectral convergence [16,20,28]. Furthermore, in the FCC method, the solution of a system of linear FDEs is discretized at Chebyshev Gauss–Lobatto (CGL) collocation points, and the discretized solution is obtained by using a proposed state transition matrix based on a fractional differentiation collocation operational matrix (FDCM). The force–displacement hysteresis curves of these models are plotted for different values of all model parameters. The coefficients of restitution (CORs) for the various fractional impact models and varying values of the system parameters are compared. Also, the initial impact force gradient, the penetration depth, and the separation depth are compared and discussed for the fractional viscoelastic models. The results show that the one-dimensional compliant impact problem can be modeled more realistically with the use of less parameters using fractional linear viscoelastic models as compared with the use of integer order linear viscoelastic models.

This paper is organized as follows. First, an overview of some basic concepts in fractional calculus is given in Section 2. In Section 3, the fractional Chebyshev collocation method is proposed for the numerical analysis of fractional-order models. Then, the one-dimensional impact problem is described and the issues with the basic integer-order viscoelastic models are listed in Section 4. Later, four aforementioned fractional viscoelastic models are introduced in Section 5 and their force–displacement hysteresis curves are obtained for different parameter values. Finally, the COR of the fractional models is compared in Section 6. The conclusion remarks are given in Section 7.

2. Preliminaries

In this section, some basic definitions of fractional calculus are recalled. Fractional differential and integral operators are defined based on a natural extension of the order of the integer order operators by their definition. The Grünwald–Letnikov (GL), Riemann–Liouville (RL), and Caputo (C) definitions are the most accepted definitions of fractional-order operators in fractional calculus.

The GL derivative is the extension of the finite difference expression for differentiation. The nth derivative of $f(x)$ is given by

$$\mathrm{D}^n f(x) = \left(\frac{\mathrm{d}_h - 1}{h}\right)^n f(x), \quad n \in \mathbb{N} \tag{1}$$

where the operator d_h and its iterates are defined as

$$\mathrm{d}_h f(x) = f(x+h) \tag{2a}$$