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A nonlinear circular ring model with rotating effects for tire vibrations

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A B S T R A C T

Rolling noise contributes significantly to the noise inside cars. This noise comes from the tire/road contact and for low frequencies (0–400 Hz), it is mainly transmitted into the cabin through structural vibrations. Thus estimating this noise requires modelling the tire vibrations by taking into account the rotating effects and the contact with rough surfaces. Concerning the model of rolling tire, a formulation of a deformable solid is constructed by using an Arbitrary Lagrangian Eulerian approach. This formulation is applied on a new simplified tire model which is a circular ring including shear stresses and nonlinear effects due to the vehicle load. This model is successfully validated by comparison with FEM results.

1. Introduction

The interior noise of vehicles has an important source coming from tires in the low frequency range, typically up to 400 Hz. For these frequencies the rolling noise is mainly due to tire vibrations. These vibrations are themselves created by the unsteady contact between road asperities and the tire tread pattern. For predicting this noise generation an accurate tire model is necessary. In the past, various tire models have been developed focusing on different aspects of the problem.

A first class of models is the two and three-dimensional circular ring models. For instance Böhm [1], Heckl [2] and Kropp [3] have modelled the tread as a circular Euler–Bernoulli beam. Sidewalls are represented by radial and tangential springs. This model takes into account the effect of the internal pressure and is linear. These circular ring models are very useful for analysing the radial vibrations of tires for low frequencies. In 2000, Périsse [4] compared the computation of the velocity of a point on the ring with its experimental value and showed a good agreement at low frequencies (below 400 Hz). Several authors developed circular ring models by adding the effect of rotation, see for instance Meftah [5], Périsse [6] and

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Campanac [7,8]. In this case one can reproduce the phenomenon of splitting of modes depending on the rotational speed. This property is also found in the work of Kim and Bolton [9]. In addition, Huang [10] has analysed the rotating ring model under a suspension system.

So, two-dimensional circular ring models allow the modelling of the dynamic behaviour of tires for low frequencies (0–400 Hz). Analytical transfer functions can be obtained to deal with contact problems. To use these models, it is necessary to identify some parameters (stiffness of the spring, bending stiffness, etc.). However, these two-dimensional circular ring models do not allow observing out of plane modes (toggle mode). That is why Eichler [11] and Gipser [12] enriched a circular model by adding a lateral stiffness to the tread and a model of rigid rim. All these models are linear and do not allow to consider, for instance, the influence of the vehicle load or nonlinear material behaviours on tire vibrations.

For higher frequencies, one needs other approaches. In 2003, Muggleton [13] modelled a tire using two orthotropic plates in traction. The tread is represented by a plate and the sidewalls are replaced by two plates. Hamet [14] has analysed the vibrations of a tire under a point force applied at the centre of the plate. This allows the construction of Green's functions of the tire used to treat the contact problem. Both models of Hamet [14] and Muggleton [13] do not take into account the effect of the curvature of the tire. Another model with two plates has been developed by Pinnington [15–17]. His model takes into account the curvature of the tire. The effect of the curvature of the tire is to couple the radial and circumferential movements and normal and tangential forces in all the vibratory responses of the tire. The dynamic equations of the tread are written for one-dimensional waves propagating around the belt and a standing wave through the tread. The effects of curvature, shear stiffness, inertia of rotation, stress, rotation velocity and the air pressure are included. The two-dimensional Mindlin plate is used to formulate this model. It is applicable to a wide band of frequencies (0–3 kHz). The difficulty of the model is still the determination of the parameters of the tread and the sidewalls. Kim and Bolton [18] have also proposed a model of rotating cylindrical shell. Similarly, based on the analytical model developed by Kropp, Wullens and Kropp [19] analysed the vibration field of a tire and the Doppler effect due to the rotation.

Because of the complexity of the tire geometry and the heterogeneous materials from which it is made, an analytical model cannot completely describe the vibrations of the tire. Moreover, the coupling between the air cavity and the tread is difficult to describe analytically. Therefore it is necessary to build numerical models of the whole set, tire/wheel/cavity, for predicting the vibratory responses of a tire. For example, one can cite the models of Takagi [20], Narasimha [21], Richards [22] and Cho [23]. Taking into account the different materials of the layers of the tire, Takagi [20] used two-dimensional finite elements. The behaviour of materials is assumed linear and the tire pressure and the rotation are taken into account. This model is valid in the frequency range [0–250 Hz].

Regarding the three-dimensional numerical models, Fadavi [24] and Brinkmeier [25] used Abaqus to model a tire. The properties of the model are identified by experimental measurements. The carcass and sidewalls are described by transverse isotropic materials. The rubber of the pads is described by a hyper-elastic material and the contact with the road is taken into account. This model is valid up to 1500 Hz. Narasimha [21] also modelled a smooth tire in contact with an obstacle and Richards [22] modelled the tire air cavity. The coupling between the fluid and the structure is included in the model. So by comparing with measurements, one sees that the model is valid up to 400 Hz. Another possibility is to use waveguides as in Waki [26] or Duhamel [27,28]. In this case only a small section of the tire needs to be computed which saves a lot of computational resources for medium and high frequencies. More precisely, concerning the coupling with the fluid, one has to consider the coupling with the internal fluid in the cavity between the tire and the wheel for loaded or unloaded tires leading to first resonances slightly above 200 Hz as measured and modelled for instance in [29–31]. This point will not be considered in this article as it has been well studied in these articles. Similarly no coupling with the external fluid will be considered, as this point has also been well studied in the past.

Often, authors have neglected the quasi-static deformation of tires and confused the stationary configuration with the configuration of reference (the configuration of reference is the initial configuration which undergoes a rigid rotation and the stationary configuration is that of the quasi-stationary regime wherein all material points deform statically under the effects of exterior loads). Under the load of the vehicle and the effect of the air pressure, the tire undergoes a nonlinear deformation. It is thus necessary to distinguish the deformed and non-deformed configurations. The deformed configuration is illustrated by the image of a crushed tire. This configuration is not known. It is obtained by solving the equilibrium equations in stationary regime. This point was for instance developed by [32–34] which used a FEM model for computing the response of tires including the quasi-static deformation and gyroscopic effects. However, these FEM models lead to heavy computations.

This paper aims at developing a nonlinear ring model to estimate the influence of these nonlinearities with a low computational cost. A first step towards the solution of this problem is a good understanding of the nonlinear behaviour of beams. In this domain, Antman et al. [35–43] have established general formulations for the behaviour of beams. Simo [44–46] described the problem of nonlinear beams with large rotations in three dimensions. These authors have also proposed a finite element method to solve the problem. Davi [47] took the Kirchhoff hypothesis to build equilibrium relationships for a thin beam. He therefore neglected the deformation due to shear stress. Applying these results Lanzo [48,42] has analysed the stability of a multi-layer model of a rubber beam and the influence of a large shear force on the nonlinear deformation of a beam.

From these ideas, we develop here a nonlinear circular ring model with a good representation of shear deformations to get a mostly analytical model able to estimate the influence of non-linearities. Note that we focus on the behaviour of the tire and not on the solution of the global problem of a tire rolling on a real road. So the contact with the road is not in the

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