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On optimal performance of nonlinear energy sinks in multiple-degree-of-freedom systems

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ABSTRACT

We study the problem of optimizing the performance of a nonlinear spring–mass–damper attached to a class of multiple-degree-of-freedom systems. We aim to maximize the rate of one-way energy transfer from primary system to the attachment, and focus on impulsive excitation of a two-degree-of-freedom primary system with an essentially nonlinear attachment. The nonlinear attachment is shown to be able to perform as a 'nonlinear energy sink' (NES) by taking away energy from the primary system irreversibly for some types of impulsive excitations. Using perturbation analysis and exploiting separation of time scales, we perform dimensionality reduction of this strongly nonlinear system. Our analysis shows that efficient energy transfer to nonlinear attachment in this system occurs for initial conditions close to homoclinic orbit of the slow time-scale undamped system, a phenomenon that has been previously observed for the case of single-degree-of-freedom primary systems. Analytical formulae for optimal parameters for given impulsive excitation input are derived. Generalization of this framework to systems with arbitrary number of degrees-of-freedom of the primary system is also discussed. The performance of both linear and nonlinear optimally tuned attachments is compared. While NES performance is sensitive to magnitude of the initial impulse, our results show that NES performance is more robust than linear tuned mass damper to several parametric perturbations. Hence, our work provides evidence that homoclinic orbits of the underlying Hamiltonian system play a crucial role in efficient nonlinear energy transfers, even in high dimensional systems, and gives new insight into robustness of systems with essential nonlinearity.

1. Introduction

The suppression of vibrational energy via transfer from the main structure to an attachment – both actively and passively – has been a lively research area since the seminal invention of the tuned mass damper (TMD) [1]. With advances in electromechanical devices, active control schemes offer the best performance in terms of vibration absorption. However, in addition to cost and energy consumption associated with active control, robustness and stability are crucial factors. Passive vibration reduction approaches include direct use or variations of linear tuned mass dampers. However, even if the tuned

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mass damper is initially tuned to eliminate resonant responses near the eigenfrequency of the primary system, the mitigating performance may become less effective due to natural mistuning of the system parameters (e.g. the varying mass of the secondary due to time varying load). In H_∞/H_2 optimization the TMD is designed such that the maximum amplitude magnification factor or the squared area under the response curve of the primary system is minimized, respectively. Analytical solutions for the H_∞/H_2 optimization of the TMD have been found [2], in the form of series solution for the H_∞ optimization and a closed-form algebraic solution for the H_2 optimization. In related work [3], an optimization problem is considered which either minimizes the kinetic energy of the host structure or maximizes the power dissipation within the absorber. Formulas for the optimal ratio of the absorber natural frequency to the host natural frequency and optimal damping ratio of the absorber were also obtained in that work. Ref. [4] deals with the analysis and optimization of tuned mass dampers by providing design formulas for maximizing the exponential time-decay rate of the system transient response. A detailed analysis is presented for the classical TMD configuration, involving an auxiliary mass attached to the main structure by means of a spring and a dashpot. Analytic expressions of the optimal exponential time-decay rate are obtained for any mass ratio and tuning condition. Then, a further optimization with respect to the latter is performed.

Lyapunov's second method has been used to minimize an integral square performance measure of damped vibrating structures subject to initial impulse [5]. Using the same approach, the closed-form solutions of optimum parameters for undamped primary structure utilizing Kronecker product and matrix column expansion are derived in Ref. [6]. Other related works include a parametric study on a TMD using steady-state harmonic excitation analysis and time-history analysis (with the El Centro and Mexico earthquake excitation signals) [7], H_∞ optimal design of a dynamic vibration absorber variant (the damping element is connected directly to the ground instead of the primary mass) for suppressing high-amplitude vibrations of damped primary systems [8], and determination of optimal absorber parameters to maximize of the primary system frequency response [9]. A simple method for choosing optimal parameters for a two-degree-of-freedom (translational/rotational) TMD has been reported in [10]. This method uses the fixed points of the frequency response functions to determine the stiffness of the TMD for a given mass. The effectiveness of TMDs in reducing the transient structural response for impulsive loadings has also been investigated in Ref. [11].

Nonlinear energy transfer between modes due to resonance has also been studied extensively [12], focusing on modal interactions and transfers from high to low frequency modes. The energy transfer phenomenon in this class of systems is essentially modal, and does not necessarily translate to one-way transfer between spatially distinct components of the system. Transition between resonances in Hamiltonian systems has been studied via geometrical and analytical methods in the past few decades [13,14]. Recently, use of active control strategies to move the system between these resonances has been explored [15–18].

Targeted energy transfers (TETs), i.e. passively controlled transfers of vibrational energy in coupled oscillators to a targeted component where the energy eventually localizes, have been a topic of great interest in the past decade [19]. The basic device is called a nonlinear energy sink (NES), which generally consists of a light mass, an essentially nonlinear spring and a viscous damper. Properly designed, the NES is capable of one-way channeling of unwanted energy from a primary system to NES over broadband frequency ranges. TET is realized through resonance captures and escapes from resonances, following (countable infinite) resonance manifolds due to the essential nonlinearity. While the phenomenon of targeted energy transfer has been extensively studied in this context [20–22], the parameter selection and optimization problem for multiple-degree-of-freedom systems is still a challenge. In Ref. [23], energy transfer initiated by an impulsive input in a single-degree-of-freedom system coupled with NES was analyzed and the optimal energy transfer phenomenon was described in terms of existence of a homoclinic orbit in a reduced phase space of the undamped (Hamiltonian) averaged (slow) system.

In this paper, we extend this analysis of Ref. [23] to a class of weakly damped multiple-degree-of-freedom systems, especially focusing on a two-degree-of-freedom system with an attached NES. We obtain the near-optimal parameters for the NES using the complexification-averaging technique and slow flow analysis. The validity of dimensionality reduction enabled by our analysis is supported by numerical comparisons between original and reduced order systems. Using a combination of perturbation analysis and simulation, we show that under assumptions of weak damping in the linear system, the homoclinic orbit picture persists in higher degree-of-freedom systems. We use Lyapunov analysis to optimize a linear TMD using a similar cost function, and obtain a semi-analytical solution for optimal parameters. Using the semi-analytical formulae, we are able to perform extensive performance comparison studies using these two classes of optimally tuned vibration absorbers.

The structure of the paper is as follows. In Section 2, we consider a two-degree-of-freedom linear system (called primary) with an attached NES. We perform a numerical study to compute various branches of periodic solutions, and obtain the frequency-energy plot. Focusing on the 1:1:1 resonance between the two masses of the primary system, and the nonlinear attachment, we perform complexification-averaging and the slow-flow averaging analysis. We elucidate the factors affecting the targeted energy transfer from the main structure to the attachment, and use another time-scale to capture the evolution of the system (super-slow flow) near the fixed point in the averaged phase space. The optimal parameters are found by analyzing the system at super-slow time scale. We provide perturbation theoretic arguments along with numerical evidence for the validity of the model. In Section 4, we describe a semi-analytical process to optimize a linear tuned mass damper attached to one and two-degree-of-freedom system. We use Lyapunov analysis to formulate the optimization problem, using energy dissipated through the attachment as the metric. In Section 5, we compare the vibration suppression performance of the two optimized attachments, i.e. the NES and linear TMD, both attached to a two-degree-of-freedom system. The results show that while NES performance is sensitive to the energy of the impulse input, it is more robust than TMD in

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