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Methodology for nonlinear quantification of a flexible beam with a local, strong nonlinearity

Christopher A. Herrera [a,b,*], D. Michael McFarland [b], Lawrence A. Bergman [b], Alexander F. Vakakis [c]

[a] General Dynamics Electric Boat, New London, CT 06320, United States
[b] Department of Aerospace Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, United States
[c] Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, United States

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ABSTRACT

This study presents a methodology for nonlinear quantification, i.e., the identification of the linear and nonlinear regimes and estimation of the degree of nonlinearity, for a cantilever beam with a local, strongly nonlinear stiffness element. The interesting feature of this system is that it behaves linearly in the limits of extreme values of the nonlinear stiffness. An Euler-Bernoulli cantilever beam with two nonlinear configurations is used to develop and demonstrate the methodology. One configuration considers a cubic spring attached at a distance from the beam root to achieve a smooth nonlinear effect. The other configuration considers a vibro-impact element that generates non-smooth effects. Both systems have the property that, in the limit of small and large values of a configuration parameter, the system is almost linear and can be modeled as such with negligible error. For the beam with a cubic spring attachment, the forcing amplitude is the varied parameter, while for the vibro-impact beam, this parameter is the clearance between the very stiff stops and the beam at static equilibrium. Proper orthogonal decomposition is employed to obtain an optimal orthogonal basis used to describe the nonlinear system dynamics for varying parameter values. The frequencies of the modes that compose the basis are then estimated using the Rayleigh quotient. The variations of these frequencies are studied to identify parameter values for which the system behaves approximately linearly and those for which the dynamical response is highly nonlinear. Moreover, a criterion based on the Betti-Maxwell reciprocity theorem is used to verify the existence of nonlinear behavior for the set of parameter values suggested by the described methodology. The developed methodology is general and applicable to discrete or continuous systems with smooth or nonsmooth nonlinearities.

1. Introduction

Systems encountered in practice generally exhibit some amount of nonlinear behavior. Often, these systems are intentionally or unintentionally simply modeled as linear, neglecting the nonlinear effects. However, important dynamical behavior inherent in certain nonlinear systems, such as the presence of more modes of vibration than the number of degrees

* Corresponding author at: General Dynamics Electric Boat, New London, CT 06320, United States.
E-mail address: caherre2@illinois.edu (C.A. Herrera).

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of freedom (due to bifurcations of nonlinear modes), is sometimes misidentified as noise or simply ignored in an attempt to reduce the complexity of the analysis. Other times, nonlinear effects are negligible in the system's regime of operation and do not require the steep analytical and computational costs associated with the development of an accurate nonlinear mathematical model.

Previous work has focused on nonlinear system identification (NSI), which involves characterization and parameter estimation of the properties of a system through mathematical analysis of experimental results. System identification, modal analysis, and reduced-order modeling (ROM) have been extensively studied for linear systems [1]. However, challenges arise when nonlinear systems are encountered, such as nonstationarity and nonlinearity in the measured responses. Yet, strongly nonlinear effects can often occur in engineering practice, such as local buckling, geometric or kinematic effects, plasticity, clearance and backlash, hysteresis, friction-induced oscillations, and vibro-impact motions. Such effects cannot be accurately modeled and analyzed by linear techniques, such as experimental modal analysis, underscoring the need for developing NSI and ROM techniques capable of characterizing broad classes of nonlinear dynamical systems. Current NSI and ROM methods are reviewed by Kerschen et al. [2,3], including the method of proper orthogonal decomposition (POD) [4]. Recently, a new NSI technique with promise of broad applicability was presented, employing empirical mode decomposition [5], under the assumption that the measured time series can be decomposed in terms of a finite number of harmonic components in the form of ‘fast,’ nearly monochromatic, oscillations that are modulated by ‘slow’ varying amplitudes [6,7]. In [8], Kurt et al. studied a linear cantilever beam with a cubic spring attached to its free end and presented an NSI method capable of identifying nonlinear modal interactions between affected cantilever modes. It was found that the strong nonlinearity mostly affects the lower-frequency bending modes and gives rise to strongly nonlinear beat phenomena, caused by internal resonance interactions of nonlinear normal modes (NNMs) [9] of the system. These internal resonances have strong energy dependence on the frequency of oscillation of the corresponding NNMs of the beam and occur at energy ranges at which the frequencies of these NNMs are rationally related. Moreover, it was found that the nonlinear effects start at a different energy level for each mode because lower modes are influenced at lower energies than higher modes. These findings will be generalized in this work.

In particular, the time history of the response of a linear cantilever beam with a local, strong nonlinearity is studied to develop mathematical techniques and quantitative and qualitative measures for identifying parameter ranges for which the system behavior is almost linear or strongly nonlinear, as well as the intermediate weakly nonlinear transitional regimes between these two cases. The only assumptions made a priori are that, in the limits of very small and very large parameter values, the system behavior is linear and the linear modes can be directly identified using linear techniques. The developed nonlinearity quantification methodology is new and advantageous because only the time histories of the system responses need to be computed, with no filtering or other preprocessing, such as Fourier transformations. The obvious drawback of this methodology is its dependence on the response, which may be recorded for a period of time in which the effects of nonlinearities are not captured completely. However, with scrupulous data collection and implementation, the developed computational tools can be successfully used to identify configurations for which nonlinear modal interactions exist based solely on time series measurements. This highlights the broad applicability of the proposed methodology, which renders it appropriate for quantifying the nonlinearities in even complex systems with smooth or non-smooth effects.

Two systems with local, strong nonlinearities are considered. First, an Euler-Bernoulli cantilever beam with a smooth stiffness nonlinearity is discussed. The nonlinear stiffness force is proportional to the third power of the displacement and has no linear term. Consequently, the local stiffness can be characterized as essentially nonlinear. A half-sine force with a half-period of 6×10^{-4} s is applied and its amplitude is varied. In the limit of very strong forcing, the restoring force generated by the spring is large and prevents the beam from deflecting significantly at the point of attachment with the nonlinear stiffness. Hence, in that limit, the response is effectively that of a fixed-pinned beam and can be modeled as such. In the other limiting case, that of weak forcing, the restoring force is almost nonexistent due to the resulting small displacements at the point of attachment, and the system can be modeled as a fixed-free beam. It follows that in the limits of strong or weak forcing, the response is nearly linear, whereas at intermediate forcing magnitudes, the response is weakly nonlinear and interesting nonlinear dynamics are realized.

The other system considered herein is similar with the important difference that its response is non-smooth. Specifically, a cantilever beam undergoing vibro-impact against very stiff stops offset by a clearance was considered. This system was considered by Mane et al. in [10]. Her work was later utilized by Kurt et al. in [11] and Chen et al. in [12]. In Mane's work, nonlinear system identification of the vibro-impact dynamics was performed by employing the assumed modes method to computationally model the response, and experimental tests to validate the computational simulations and predictions. Indeed, much of the work by Mane et al. served as the basis for validation of the modeling methods used in this study. For example, the modeling of vibro-impact of the cantilever beam as piece-wise linear by incorporating a linear spring and dashpot attachment offset by a clearance was used in that work. In another study, Andreaus et al. [13] also modeled impact of a cantilever beam using this same piece-wise linear technique with a modal superposition model and verified the model experimentally close to and away from resonance. Other modeling methods were also considered, such as regarding impact as a prescribed displacement boundary condition, described by Quinn in [14], but these proved difficult to implement in the present systems. Thus, the method in [10] was used with suggestions for identifying the contact and separation times inspired by [15]. In that work, Yigit et al. studied a radially rotating beam undergoing vibro-impact. The simulation time step was reduced until the system was within a prescribed tolerance of the impact point. In this study, the varying parameter for the vibro-impact beam is the clearance between the stops and the beam at static equilibrium. As in the first of the systems