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High-order full-discretization method using Lagrange interpolation for stability analysis of turning processes with stiffness variation

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ABSTRACT

In turning processes, chatter is an unstable vibration which adversely affects surface finish and machine tool components. *Stiffness variation* (SV) is an effective strategy for chatter suppression by periodically modulating the stiffness around a nominal value. The dynamics of SV turning is governed by a time periodic *delay differential equation* (DDE) where the *time-period/time-delay ratio* (TPTDR) can be arbitrary. Recently, first-, second- and higher-order *full-discretization methods* (FDMs) have been reported as a popular class of methods for milling stability prediction. However, these FDMs can only deal with time periodic DDE where the TPTDR equals one. In this paper, two *high-order FDMs using Lagrange interpolation* (HLFDMs) are proposed for stability analysis of SV turning. On each discrete time interval, the time delay term is interpolated by the second-degree Lagrange polynomial, and the time periodic term is linearly interpolated. The state term is approximated using linear interpolation and second-degree Lagrange polynomial interpolation, achieving the first- and second-order HLFDM, respectively. Finally, the transition matrix over a single period is deduced for stability analysis via the Floquet theory. Benchmark examples of damped delay Mathieu equations are used to verify the proposed algorithm, which demonstrates that HLFDMs are highly efficient and accurate. In addition, the second-order HLFDM is used to investigate the effects of SV amplitude and frequency parameters. These results provide theoretical insights for the selection of SV parameters.

1. Introduction

Chatter is a form of self-excited, unstable vibration in almost all machining processes. Chatter can be beneficial in cases such as self-excited, parametric, or assisted vibratory drilling [1–3]. However, it often has negative effects on machining quality, and is thus a significant problem that remains to be solved. Regenerative chatter, as the most common form of chatter, usually occurs with excessive vibration between the tool and the workpiece. As reviewed in [4,5], the catastrophic nature of chatter creates numerous problems like poor surface finish, excessive noise, breakage of machine tool components, reduced tool life and productivity. The need for better chatter suppression techniques is ever increasing due to demands in manufacture industries for higher productivity and precision.

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Active chatter suppression methods have attracted increasing interest with technological advances in computers, on-machine sensors and actuators [4,5]. Among active methods, disturbing the regenerative effect can be highly efficient by breaking the periodicity required for the full development of chatter. Either process or structural parameters might be used to this end. Recently, *stiffness variation* (SV) [6–10] has proven to be an efficient strategy for chatter suppression by modulating the stiffness periodically. Researchers have studied the mechanism of SV turning from the perspective of the indispensable condition of resonance [6], Nyquist diagram [7,8] and energy accumulation [9]. Approximating the time delay term via Taylor series, Yao *et al.* [10] analyzed SV turning using an averaging method. However, the effects on the turning stability induced by this approximation were not discussed in their work. In studies [8–10], the frequency of the SV was generally set to 1-6 Hz without considering the relation to the delay or to the chatter frequency. Nevertheless, frequencies were selected heuristically without theoretical support. Despite its substantial value, SV has not yet gained widespread acceptance in industrial machining applications. One primary reason is the inadequacy of theoretical study on the selection of SV parameters, specifically the variation of frequency and amplitude.

The dynamics of SV turning is governed by a time periodic *delay differential equation* (DDE) where the *time-period/time-delay ratio* (TPTDR) could be arbitrary. Various techniques exist that focus on stability analysis of time periodic DDEs. The *semi-discretization method* (SDM) developed by Insperger and Stépán [11], is an efficient numerical method to analyze the stability of time periodic DDEs. In the SDM, the delayed term was approximated as a weighted linear combination of the delayed discrete values while the time periodic coefficients were approximated by piecewise constant functions.

Using first- and second-order approximations of the delayed term, the first- and second-order SDMs [12] were developed to achieve faster convergence than the zeroth SDM in [11].

In contrast to the SDM, Ding et al. [13] proposed the *full-discretization method* (FDM) to predict milling stability, which has high computational efficiency and fast convergence rate. In the FDM, the delay terms, periodic coefficients and actual time-domain states are simultaneously discretized, which leads to less computational cost than the SDM. Insperger [14] then compared the FDM with the zero-order SDM and the first-order SDM in detail. The results show that the local discretization error of the FDM is similar to that of the zeroth-order SDM. To improve the performance of the FDM, the second-order FDM [15] and the third-order FDM [16] are presented sequentially by interpolating the state term with higher-order polynomials. Moreover, the FDM beyond the third order are explored by Ozoegwu et al. [17].

It is noteworthy that all existing FDMs mentioned above are developed for time periodic DDEs when the TPTDR equals one, however, they are unable to deal with applications of other TPTDRs. Recently, Liu et al. [18] propsed a new *Hermite-interpolation FDM* (HFDM) to handle the time periodic DDE when the time delay is unequal to the time period. However, the accuracy of the HFDM decreases when the time period and the time delay are not integer multiples.

In this paper, two *high-order FDMs using Lagrange interpolation* (HLFDMs) are proposed for stability analysis of SV turning governed by time periodic DDEs with arbitrary TPTDRs. The efficiency and accuracy of the proposed algorithms are verified by bench examples of damped delay Mathieu equations. Then, the second-order HLFDM is employed to analyze stability of SV turning. The effects of SV parameters, i.e. the variation amplitude and frequency, on turning stability are investigated.

The remainder of this paper is arranged as follows. Section 2 introduces the stability model of SV turning. In Section 3, the first-order and second-order HLFDM are presented in detail. Benchmark examples of damped delay Mathieu equations are utilized to verify accuracy and computational efficiency of the proposed method in Section 4. In Section 5, the proposed algorithm is applied for stability analysis of the turning process with the SV. Finally, Section 6 concludes this paper.

2. Modeling of turning with stiffness variation

2.1. Turning process

The mathematical model considering a *single degree of freedom* (SDoF) turning process with the flexible tool and relatively rigid workpiece is shown in Fig. 1. The model incorporates various forces acting on the physical system like the inertia force, damping force, spring force and cutting force [19]. The model is presented by considering a sharp tool with only the cutting force in feed direction acting in the system. The equation of motion of the dynamic system can be modeled in the feed direction as

$$\ddot{y}(t) + 2\xi\omega_n\dot{y}(t) + \omega_n^2 y(t) = \frac{k_d a_p}{m_t}\left[h_0 + y(t-\tau) - y(t)\right] \tag{1}$$

where k_d is the cutting force coefficient in each cutting area unit, a_p is the depth of cut, m_t is the modal mass of the tool, h_0 is the nominal cutting thickness, ξ is the damping ratio and ω_n is the natural frequency. The modal stiffness is ω_n^2 in $(\text{rad/s})^2$. The right side of (1) represents the cutting force in the feed direction. Stability analysis is carried out on the equation of motion in perturbations, which is similar to (1), without the constant h_0.

2.2. Stiffness variation

As mentioned above, the SV is an effective technique, which actively suppresses chatter by varying the stiffness