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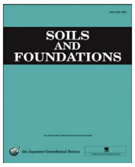

Characterisation of the ultimate particle size distribution of uniform and gap-graded soils

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Abstract

The ultimate particle size distribution of uniform and gap-graded soils is examined on specimens of carbonate sand that were subjected to large strains in a ring shear apparatus. The gap-graded soils were seen to retain a memory of their initial grading even at large strains. The particle size distributions were plotted in double logarithmic graphs either by mass or by number computed assuming different shapes. It was not possible to find linear subsets of the data, and since the samples were found experimentally to have converged to an ultimate grading, this suggests that the initial bimodal distribution prevented reaching an ultimate fractal distribution. Plots of the probability density functions of the particle sizes before and after shearing show the evolution of the gap-graded soils from a bimodal to a multi-modal distribution. This is accompanied by an evolution of the shape of the particles, visible in microphotographs and projections of the grains before and after test.

Keywords: Sand; Particle size distribution; Particle shape; Grain crushing; Fractal

1. Introduction

Most soils subjected to compression to high stresses or shearing to large strains suffer particle breakage. The existence of an ultimate grading for soils has been suggested from experimental data, for example by Coop et al. (2004) for uniform sands sheared to large strains, although there were different gradings for different normal stresses, or by Altuhafi et al. (2011) for a natural subglacial till. Turcotte (1986) reported that many granular geomaterials resulting from weathering or fragmentation follow a power law frequency distribution of sizes, creating fractal sets (Mandelbrot, 1982). This implies that the probability of any size range to break is the same (scale invariance). This concept has been increasingly used in soil mechanics in models for particle breakage (e.g. McDowell and Bolton, 1998; Einav, 2007a; Russell, 2011) or to characterise ultimate particle size distributions (e.g. Altuhafi et al., 2011). It is not clear however that the ultimate grading of atypical soils such as bimodal soils would satisfy fractality.

Two approaches have been reported in the literature, a mass-based approach that uses sieving test data (e.g. Coop et al., 2004; Altuhafi and Coop, 2011), and a number-based approach that computes the number of soil particles from the mass, generally by assuming a constant shape of particles (e.g. Tyler and Wheatcraft, 1989;

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Hooke and Iverson, 1995; Altuhafi and Baudet, 2011). In the field of soil science, Perfect et al. (1992) showed that for silt loam soils with particles sizes ranging between 0.5 and 30 mm the fractal dimensions computed by number- and mass-based approaches were the same. The traditional way to describe fractals is by a power law between number and size (Mandelbrot, 1982):

$$N \propto r^{-D} \tag{1}$$

where N is the number of objects with a linear dimension greater than r; the exponent D is defined as the fractal dimension. Turcotte (1986) proposed that the size distribution resulting from fragmentation can be expressed as:

$$N(>m) = Cm^{-b} \tag{2}$$

where $N(>\text{m})$ is the number of fragments with a mass greater than m, and C and b are constants, b being equivalent to the fractal dimension. For a material of constant density (or constant specific gravity), the mass is proportional to the volume ($m \propto r^3$), so if the volume is taken simply as r^3 (i.e. no shape is implied), by replacing into (2) and comparing with (1) we obtain:

$$D = 3b \tag{3}$$

The two power law distributions (1) and (2) have been considered equivalent, and several earth scientists (e.g. Sammis et al., 1987; Hooke and Iverson, 1995; Benn and Gemmell, 2002) as well as soil scientists (e.g. Kozak et al., 1996; Grout et al., 1998) have determined the fractal nature of soil particle size distributions from a double logarithmic plot of number of particles against size. The fractal dimensions computed in this manner range between 2 and 3 (Turcotte, 1986; Kozak et al., 1996), with some exceptions: for example Hartmann (1969) reported values of 1.89 for artificially crushed quartz and 3.54 for ash and pumice. Kozak et al. (1996) noted that while (1) can be used to generate distributions with values of D larger than 3, these distributions contain a majority of fines and do not represent the result from pure fragmentation modelled by Turcotte (1986). It is not uncommon to find such high values in glacial tills which have been created by a mixture of intense crushing and abrasion during shearing underneath glaciers (e.g. Altuhafi et al., 2010; Altuhafi and Baudet, 2011). By comparison, the mass-based approach makes direct use of the sieving test data. For a fractal distribution of particle sizes, the slope of the cumulative mass distribution versus size in a double logarithmic plot is (3 – D) (e.g. Bird et al., 2000). The fractal dimensions determined using this method are around 2.5 for pure sands (e.g. McDowell and Bolton, 1998; Millán et al., 2003; Coop et al., 2004).

Earlier work on the particle size distribution of soils used probabilistic models with lognormal distributions (e.g. Epstein, 1948), which were thought to fit data better than fractals. Millán et al. (2003) showed how using a piecewise fractal model to fit different fractal sets in different size ranges may be better suited to describe granular soils. Miao and Airey (2013) tested a uniform and gap-graded soil (40% small grains) in compression and shearing and found that two fractal dimensions could be defined over two size ranges for each soil, with a cut-off at 75 microns for the uniform soil, which corresponds to the silt sieve size, and at about 150 microns for the gap-graded soil, which corresponded to the size of the small grains. They described the distributions as multifractal, which is perhaps misleading as this should refer to particle size distributions with continuously changing fractal dimensions. Huang and Bradford (1992) defined distributions with distinct fractal subsets as “pseudo-fractals.

Zhang and Baudet (2013) found that gap-graded soils tend to retain the memory of their initial distribution even after compression to high stress, so that the grain size distribution after testing shows a “knee” corresponding to the size of small particles in the gap-graded soil (example shown later in Fig. 3). Zhang and Baudet (2015) found that the probability density functions of grain sizes of a uniform and a gap-graded carbonate sand exhibited several peaks over distinct size ranges after shearing. They explored whether there is a correspondence with the distinct fractal sets determined from the number-based distribution but could not reach any firm conclusion. This paper examines the different ways of characterising the ultimate particle size distribution by using data from tests on uniform and gap-graded soils after shearing to very large strains. The data are analysed as cumulative distribution functions by mass and by number, and as probability density functions. A brief assessment of their fractality is given. Additional information is given from analyses of the distribution of particle shapes before and after shearing using probability density functions combined with micrographs and projections of grain images.

2. Materials and testing procedures

The tests were carried out on biogenic carbonate sand (CS) from the South China Sea comprising mainly mollusc and foraminifera shells. The sand was first separated into different uniform sizes by mechanical sieving. Six sizes of grains were selected, $d_{\text{small}} = 0.063$–0.15 mm, 0.15–0.212 mm, and 0.212–0.3 mm for the small particles, and $d_{\text{large}} = 0.6$–1.18 mm, 1.18–2.0 mm, and 2.0–2.36 mm for the large particles. Specimens were prepared with a ratio of large to small particles R kept approximately constant, with R = 8.35, 8.78 and 8.52 for the smaller, medium- and larger-sized samples respectively so the effect of size rather than ratio of sizes could be highlighted. The specimens were prepared at a designed initial grading by mixing small and large particles in exact proportion from 20 to 60% small grains (SG) content. A total of ten tests were carried out, as summarised in Table 1.

In order to study the ultimate particle size distribution of soil by fractal analysis, it is necessary to continue crushing the soil grains until a stable grading is reached. The ring shear apparatus allows reaching very large strains and obtain significant breakage in the soil. Coop et al. (2004)

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