Structural Safety 66 (2017) 38–50

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Cross-entropy-based adaptive importance sampling for time-dependent reliability analysis of deteriorating structures

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A R T I C L E I N F O

Article history:
Received 19 October 2015
Received in revised form 7 July 2016
Accepted 17 December 2016

Keywords:
Time-dependent reliability
Adaptive importance sampling
Cross-entropy
Gaussian mixture
Monte Carlo simulation
System reliability

A B S T R A C T

Time-dependent reliability analysis of deteriorating structures is important in their performance evaluation and maintenance. Various definitions and methods have been used by researchers to predict the time-dependent reliability of structures. In the present study, these methods are first critically reviewed and examined. Among these methods, the stochastic-process-based method is theoretically the most rigorous but also computationally the most expensive. To facilitate the wide application of the stochastic-process-based method in complex problems, an efficient importance sampling method is then proposed in this paper. The proposed method includes a number of improvements formulated to enhance the efficiency and robustness of an existing method proposed by Kurtz and Song, leading to more efficient solutions of time-dependent reliability problems of structural systems with multiple important regions. The validity and efficiency of the new method is demonstrated through three numerical examples.

1. Introduction

Structures experience deterioration and varying load effects during their service life. Structural reliability during the structural lifetime is in essence a time-dependent property that cannot be well represented within the framework of time-independent reliability. Therefore, time-dependent reliability analysis has been used by various researchers to evaluate structural safety throughout the entire service life of structures or civil infrastructure systems. Different researchers have often adopted different methods in their analysis, and these methods can, in some cases, lead to dramatically different results. The theoretically rigorous methods (e.g. the stochastic-process-based method) for time-dependent reliability analysis usually involve tedious computation, which hinders their application to complex reliability problems with multiple important regions (e.g. time-dependent reliability of structural systems). The objective of this paper is twofold. First, existing methods for time-dependent reliability analysis are critically reviewed. Secondly, a new and efficient sampling method is proposed to facilitate the wide application of the stochastic-process-based method in complex time-dependent reliability problems.

During the service life of a structure, its resistance is likely to deteriorate due to factors such as material degradations (e.g. steel corrosion), damage from overloading, and natural disasters. In addition, the load that a structure has to resist may change significantly over time. Therefore, the performance function is a time-dependent function as follows:

$$g(t) = R(t) - S(t) \quad (1)$$

where $R(t)$ and $S(t)$ are the time-dependent resistance and load effect, respectively. In order to evaluate structural safety during the entire service life, various methods have been proposed [1]. Among these methods, the simplest one is the point-in-time method, which discretizes the time-dependent problem into a series of time-independent reliability problems [1]. In other words, the change of probability of failure, namely the probability of $g(t) < 0$, is characterized by a series of failure probabilities at time points and is calculated by a conventional time-independent reliability approach, e.g. the first order second moment (FOSM) method, first order reliability method (FORM) or Monte Carlo simulation (MC) [2–5]. The selection of time points is usually arbitrary, and failure at these time points is assumed to be independent of each other and memoryless. The advantage of this method is obvious. Time-dependent reliability problems are converted into time-independent problems, and the computation is relatively simple and efficient. Therefore, this method, referred to herein as the point-in-time method without failure memory, has been used by many researchers in the time-dependent reliability analysis of complex problems [2–5]. However, it should be noted that the probability of failure calculated this way

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http://dx.doi.org/10.1016/j.strusafe.2016.12.006
0167-4730/

cannot closely approximate time-dependent failure probability. For example, if Monte Carlo simulation is used to determine failure probability at a specific time point, a simulation case that has failed at time t_i may become safe again at t_{i+1}. Therefore, this method may overestimate structural reliability dramatically. A numerical example to illustrate this issue is provided later in this paper.

In order to improve the conventional point-in-time reliability method, some researchers proposed to keep track of the failure of sample cases within the service life in Monte Carlo simulation [6]. The sample cases that are labeled as "failed" will not participate in the analysis at the following points-in-time, and, therefore, they will not become safe again. In the present study, this method is referred to as the point-in-time method with failure memory. In this method, the points-in-time are no longer arbitrary. Instead, they are usually defined as those epochs at which the live load is updated [6]. For instance, for the time-dependent reliability analysis of bridges, the maximum vehicle load is updated annually according to traffic surveys so that the number of points-in-time is equal to the number of service years. Specifically, n_{MC} samples of time-independent variables (e.g. dead load and initial resistance) are first generated. Other time-dependent variables are generated and updated at each epoch of interest. At each point-in-time, the performance function is evaluated and the failure cases identified are appropriately labeled and eliminated from the subsequent analyses. The point-in-time method with failure memory has been used to evaluate the time-dependent reliability of deteriorating reinforced concrete (RC) structures [6–8].

The point-in-time method with failure memory can be further simplified to the following equation

$$P_f(t_L) = 1 - (1 - P_{f1})^{t_L} \approx t_L P_{f1} \tag{2}$$

where t_L is the service life in time units; and P_{f1} is the probability of failure in one time unit; by assuming that: a) the structural resistance is a time-independent random variable (no structural deterioration); and b) the probability of failure at each point-in-time is independent of each other [1]. Eq. (2) plays an important role in the calibration of design guidelines [9] and also has been used by researchers in time-dependent reliability analysis [10]. Herein, this method is referred to as the time-integrated method [1].

Even though the point-in-time method with failure memory and the time-integrated method are more realistic than the point-in-time method without failure memory, they imply that the number of live load events during the time $[0, t]$ is deterministic. However, in reality this number is random; that is, the arrival of live load events is actually a discrete stochastic process [1]. In addition, the time-integrated method is based on the assumption of no resistance deterioration, which makes them inappropriate for deteriorating structures. The point-in-time method with failure memory can be improved by assuming that the arrival of live load events follows a discrete stochastic process (e.g. Poisson process and Bernoulli process [11,12]). The time-dependent failure probability can then be computed using the conditional probability theory as follows:

$$\begin{aligned} P_f(t|\mathbf{x}_c) &= \sum_{k=1}^{\infty} \Pr[g(\tau; \mathbf{x}_v(\tau), \mathbf{x}_c) < 0, \tau < t | \mathbf{x}_c, k] \Pr[N(t) = k] \\ P_f(t) &= \int_{\Omega} P_f(t|\mathbf{x}_c) f_{\mathbf{X}_c}(\mathbf{x}_c) d\mathbf{x}_c \end{aligned} \tag{3}$$

where $N(t) = k$ represents the situation that k load events have occurred prior to time t; $\mathbf{x}_v$ and $\mathbf{x}_c$ are the time-dependent and the time-independent variables respectively; $f\mathbf{x}_c$ is the probability density function (PDF) of $\mathbf{x}_c$; and τ is any time prior to time of interest, t. This method was first proposed by Mori and Ellingwood [13] and has thereafter been extensively used in the time-dependent reliability analysis of deteriorating infrastructures [14–17]. In the most general sense, both the resistance and the load effect are continuous stochastic processes. Therefore, a time-dependent reliability problem amounts to a problem of determining the first-passage probability [1]. Unfortunately, analytical solutions for the first-passage probability only exist for some specific processes. In most cases, computationally expensive numerical evaluation is required. Determination of first-passage probabilities has mainly been used in stochastic dynamics where the crossing rate, an important ingredient for calculating the first-passage probability [1], is evaluated analytically and/or numerically [18–20]. For the time-dependent reliability analysis of deteriorating structures, which involves a much longer time scale, this general method has been attempted by only a few researchers [21–22]. In such problems, it can be shown that this general method reduces to Mori and Ellingwood's [13] method when the live load is a Poisson process. Therefore, in the present study, this general method and Eq. (3) are considered to belong to the same method and are referred to as the stochastic-process-based method.

Despite the rigorous nature of the stochastic-process-based method, this method also has certain limitations. From Eq. (3), it can be seen that the computation of $P_f(t)$ requires the determination of the conditional failure probability $P_f(t|\mathbf{x}_c)$, which is usually calculated using a numerical integration method [13]. This implies that the performance function cannot be very complex. The performance function of a deteriorating structure, fortunately, can usually be simplified to either Eq. (1) or the following equation:

$$g(t) = R(t) - S_D - S_L(t) \tag{4}$$

where the time-independent dead load effect S_D is isolated from the time-dependent load effect. The integration in Eq. (3) is usually carried out using a simulation method. Adaptive importance sampling has been used to enhance the computational efficiency [13].

Although the conventional adaptive importance sampling method can reduce the computational burden for some simple problems [1], the stochastic-process-based method still faces computational difficulties, especially for those problems involving multiple random variables and important regions, e.g. in system reliability problems. This is mainly because Mori and Ellingwood [13] adopted a unimodal sampling function that cannot generate samples efficiently when the actual region of importance is multimodal. Therefore, for the time-dependent reliability analysis of structural systems, both the point-in-time and the time-integrated methods are sometimes used to save computational time. However, as stated previously, the time-dependent reliability computed this way is usually non-conservative and inconsistent with that with the more rigorous stochastic-process-based method. To facilitate the application of the stochastic-process-based method in complex problems, a cross-entropy-based adaptive sampling method using Gaussian mixtures is proposed in this paper. This new method is capable of coping with multiple important regions and, hence, can significantly increase the efficiency of the stochastic-process-based method in time-dependent system reliability problems. The method is an extension of Kurtz and Song's [23] method for time-independent reliability problems to time-dependent domains. The proposed method also includes a number of improvements formulated to adapt to the unique features of time-dependent reliability problems and enhance the efficiency and robustness of the original method developed in [23].

In the following sections, the cross-entropy-based adaptive importance sampling method is first introduced in Section 2. Application of this method in time-dependent reliability analysis is next presented in Section 3. A number of measures to improve the efficiency and robustness of Kurtz and Song's [23] method are also presented in this section. Numerical examples are given in Section 4 to illustrate the necessity of using the stochastic-process-based method and the efficiency of the new method.