Structural Safety 66 (2017) 106–117

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

On characterizing spatially variable soil Young's modulus using spatial average

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ARTICLE INFO

Article history:
Received 10 January 2016
Received in revised form 1 March 2017
Accepted 1 March 2017

Keywords:
Spatial variability
Young's modulus
Three dimensions
Random field
Deformation

ABSTRACT

The purpose of this study is to investigate whether the effective Young's modulus (E_{eff}) for a spatial variable soil mass can be strongly correlated to any spatial average. The spatially variable Young's modulus of the soil mass is modeled as a stationary lognormal random field, and the E_{eff} of the soil mass is simulated by random field finite element analysis. Spatial averages are calculated from the input random field. If a strong correlation exists, it is possible to replace a random field analysis by a simpler random variable analysis. Two classes of problems are considered: a soil cube subjected to displacement-controlled compression and a footing problem. For the soil cube problem, E_{eff} is found to be strongly correlated to a suitable spatial average. However, for the footing problem, only the statistics (mean and standard deviation) of E_{eff} can be well approximated by a suitable spatial average, but the correlation is not strong. It is possible that the two classes of problems behave differently because the finite elements in the soil cube are mobilized uniformly, whereas those in the footing problem are mobilized non-uniformly. This leads to a weighted spatial average model that applies a different weight on the log modulus of each finite element over the domain being averaged.

1. Introduction

The spatial variability of soil parameters has profound impact to the behavior of a geotechnical system, and a stationary random field model has been adopted to model such spatial variability in the literature. A stationary random field is typically characterized by its mean (μ), coefficient of variation (COV = standard deviation/mean), and scale of fluctuation (SOF). The COV quantifies the magnitude of the oscillation around the mean, whereas the SOF measures the distance within which the spatial variation is significantly correlated [19]. The impact of the spatial variability in the soil Young's modulus (E) on foundation settlements has been widely studied [16,14,6,7,11,8,13,17,1,2,3]. For foundations on soils with isotropic SOFs, an important observation made in Fenton and Griffiths [6,7] is that the effective Young's modulus (E_{eff}) has statistics (mean and COV) similar to the statistics of the geometric average (E_g) over a prescribed domain under the footing:

$$\ln(E_g) = \frac{1}{D}\int_D \ln[E(x,y,z)] \cdot dx \cdot dy \cdot dz \quad (1)$$

where E(x, y, z) is the E value at location (x, y, z); D is the averaging domain. Here, the term "effective Young's modulus" refers to the Young's modulus actually "felt" by the foundation. To be more specific, the settlement of a rigid shallow foundation on a homogeneous soil mass with E_{eff} will be identical to the settlement on the spatially variable soil mass. The process of reducing spatially variable Young's modulus into E_{eff} is also called "homogenization" in the literature [10,9,15]. The observation $E_{eff} \approx E_g$ holds for two-dimensional (2D) scenarios [6] and also for 3D scenarios [7]. For foundations on layered soils, Fenton and Griffiths [7] argued that E_{eff} has statistics similar to the statistics of the harmonic average (E_h) for horizontal layers:

$$\frac{1}{E_h} = \frac{1}{D}\int_D \frac{1}{E(x,y,z)} \cdot dx \cdot dy \cdot dz \quad (2)$$

and similar to the statistics of the arithmetic average (E_a) for vertical layers:

$$E_a = \frac{1}{D}\int_D E(x,y,z) \cdot dx \cdot dy \cdot dz \quad (3)$$

It is important to emphasize that E_{eff} is determined from the deformation response of a random finite element analysis (i.e., it is an output of a boundary value problem such as a shallow foundation applying pressure on top of a semi-infinite soil domain). In contrast,

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http://dx.doi.org/10.1016/j.strusafe.2017.03.001
0167-4730/

these conventional spatial averages (E_a, E_g, and E_h) are calculated from the input random field describing the spatial distribution of the Young's modulus over the semi-infinite soil domain (i.e., they are inputs unrelated to the boundary value problem).

The similarities reported in Fenton and Griffiths [6,7] are in the statistics of E_{eff}. It is not clear whether E_{eff} is strongly correlated to the spatial average. There is no theoretical reason for this correlation to exist between an output (E_{eff}) and an input (spatial average) quantity. Note that the similarity in the statistics does not imply strong correlation, e.g., two random variables can have same statistics and yet be completely uncorrelated. A more general way of presenting this issue is to ask if an input random variable can be defined based on a conventional spatial average such as E_g, E_h, and E_a such that it is identically distributed *and fully correlated* (correlation coefficient = 1) to the output random variable E_{eff}? Given that E_{eff} is the output of a boundary value problem, a related issue is whether this conventional spatial average can be defined so that it is independent of the boundary value problem. The practical importance of these issues are quite obvious. If the answer is yes for the first issue, it will be possible to simplify a random finite element analysis involving a random field to a random variable problem which is less costly and perhaps more importantly, make probabilistic design more accessible to engineers. If the answer is yes for the second issue, a non-problem specific solution exists.

For a 2D square specimen subjected to 1D displacement-controlled compression (similar to oedometer test), Ching et al. [4] found a stronger conclusion: not only the statistics of E_{eff} can be well approximated by a suitable conventional spatial average but E_{eff} can also be strongly correlated to this spatial average (correlation coefficient close to 1). That is, for their 2D soil specimen undergoing 1D compression, the answer is yes. This is a rather remarkable outcome given an output (E_{eff}) can be "predicted" by an input (spatial average)! It is natural to ask the following question: is it true that the E_{eff} for the footing problem is strongly correlated to some suitable conventional spatial average as well? Note that the footing problem is more complicated than the 2D soil specimen. The simulation results presented in this paper show that this is not true. For the footing problem, only the statistics of E_{eff} can be well approximated by a suitable conventional spatial average, but the E_{eff} is not strongly correlated to this spatial average. That is, for the footing problem, the answer is no.

With the above past studies in mind, this paper seeks to clarify three new aspects, with a focus on understanding the effectiveness of applying conventional spatial averages to estimate the numerical value of E_{eff} in a 3D setting:

1. Demonstrate that the E_{eff} for a square specimen subjected to displacement-controlled compression is strongly correlated to a suitable spatial average. In particular, this demonstration will be conducted on a 3D soil cube in an attempt to generalize the 2D results presented by Ching et al. [4]. Moreover, it will be shown that this strong correlation is insensitive to whether stress is uniform within the soil cube as long as a proper spatial averaging model is chosen.
2. Demonstrate that the E_{eff} for the 3D footing problem is not strongly correlated to any conventional spatial average. Only its statistics can be well approximated.
3. Explore possible explanations for why the two classes of problems behave so differently. One possible explanation is illustrated: whether all finite elements are mobilized uniformly or not. The finite elements in the soil cube subjected to displacement-controlled compression are mobilized uniformly, hence a conventional spatial averaging model works well (in the sense that E_{eff} is strongly correlated to the spatial average). However, the finite elements in the footing problem are mobilized non-uniformly. A spatial averaging model that cannot accommodate such non-uniform mobilization cannot work well, even though the spatial average can have statistics similar to E_{eff}.

2. 3D soil cube under investigation

2.1. Random field model

Consider a 3D spatially variable soil mass with size $L \times L \times L = 10 \times 10 \times 10$ (Fig. 1). No unit is specified for the dimension L because all dimensions will be normalized by SOF. The Young's modulus, denoted by E(x, y, z), is modeled as a stationary lognormal random field with inherent mean = μ and inherent coefficient of variation = COV. No unit will be specified for μ because all statistical properties of the soil mass will be later normalized by μ. To define the correlation structure between two locations with horizontal distances = (Δx, Δy) and vertical distance = Δz, the single exponential auto-correlation model is considered [19,20]:

$$\rho(\Delta x,\Delta y,\Delta z) = \exp(-2|\Delta x|/\delta_x - 2|\Delta y|/\delta_y - 2|\Delta z|/\delta_z) \quad (4)$$

where δ_x, δ_y and δ_z are the SOFs in (x, y, z) directions, respectively, for the ln[E(x, y, z)] random field. A 3D stationary lognormal random field E(x, y, z) can be simulated by taking the exponential of a 3D stationary normal random field ln[E(x, y, z)] with mean = $\lambda = \ln[\mu/(1 + COV^2)^{0.5}]$ and variance = $\xi^2 = \ln(1 + COV^2)$. In this study, the local average for each finite element is taken to be E = geometric average for E(x, y, z) over the element because there is an analytical expression for local geometric average, and this local average can be simulated quite readily using the Fourier series method [12,5]. Fig. 1a shows a realization of the E random field with $\delta_x = \delta_y = \delta_z = 1$. The light region refers to low values of E, whereas the dark region refers to high values of E. The Poisson's ratio (ν) is not modeled as a random field but is taken to be a constant ($\nu = 0.3$) because these two scenarios (random field ν and constant ν) produce comparable results [6,7,4].

2.2. Finite element model

The $10 \times 10 \times 10$ cubic domain is modeled by the finite element (FE) mesh shown in Fig. 1. Each FE element has size = $0.4 \times 0.4 \times 0.4$ and is with eight nodes. In total, there are $25 \times 25 \times 25 = 15{,}625$ FE elements. Each FE element follows isotropic elasticity with E = its *local* geometric average and $\nu = 0.3$. The commercial FE package ABAQUS is adopted.

2.3. Simulation of effective Young's moduli

For each realization of the E random field, three random field finite element analyses that simulate an overall 1D compression in the (x, y, z) directions are conducted. Let us consider the x direction as an example. As shown in Fig. 2, the nodes on Face 1 are constrained in a "tied freedom" manner [15] so that all nodes on Face 1 are subjected to a uniform x-displacement of 0.1. Face 2–6 are constrained by rollers (e.g., for Face 3, y-displacement is not allowed). The predominant strain in the soil cube should be in the x-direction. The other strain components, ε_y, ε_z, ε_{xy}, ε_{xz}, and ε_{yz}, may not be zero in each element due to spatial variability, but they are likely to be minor. The boundary condition described above is similar to that imposed in an oedometer test. The effective Young modulus in the x direction, denoted by $E_{x,eff}$, can be deduced from the deformation response of the soil cube. The stress at Face 1 is not uniform because a uniform displacement boundary is prescribed. The "overall" σ_x is equal to the arithmetic average of σ_x over Face 1. Because the overall strains are $\varepsilon_x = 0.1/10 = 0.01$ and