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Free flexural vibration of symmetric beams with inertia induced cross section deformations

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ABSTRACT

Beams with large thin-walled cross sections are not generally following the classical beam theories such as Euler-Bernoulli and Timoshenko theories. In free vibration, the cross section is deformed by the inertia induced body loads. These deformations may have significant effect on the beam modal frequencies, especially in applications involving non-structural masses. This paper presents a method to include the effect into vibration modal results obtained by the classical beam theories. Generalized mass and stiffness of the classical results are modified according to kinetic-, and strain energies of the cross section deformation. The method is validated in typical engineering case studies against fine mesh Finite Element Method and excellent agreement is found.

1. Introduction

Beams with thin-walled cross sections are used in several fields of structural engineering including applications in marine, aerospace and bridge structures due to their excellent stiffness to weight ratio. Several models have been developed for modal analysis of these thin-walled beams. Euler-Bernoulli beam model [1] is accurate for slender beams with small cross section dimensions relative to length of flexural waves of the studied vibration mode. Due to its simplicity and availability on commercial Finite Element software, the model is still widely used in practical engineering. The shear-deformations can be accounted by Timoshenko beam theory [2–4] that provides significantly more accurate modal frequencies in beams that are deep relative to the length of flexural waves of the studied vibration mode. Drawback of practical applicability of Timoshenko's beam model has been definition of shear correction factors for different cross sections; see for example Refs. [5–8]. However, these methods cannot account the influences of local deformations that occur solely on the beam cross-section plane.

Effects of inertia induced cross section deformations have been studied by NACA in the 1950s and 60s. The phenomenon was observed in vibration tests of box beams [9] and the effect has been analyzed for box beams in Ref. [10]; monocoques in Ref. [11]; angled sections in Ref. [12] and channel sections in Ref. [13]. Several bridge beams were studied in Ref. [14] by including cross section deformation into equations of motion. Generalized beam theory has been developed to take all the above-mentioned effects elegantly into account inside the beam formulation [15]. It has been applied for vibration problems for example in Ref. [16]. Carrera et al. have applied unified formulation [17] to Finite element vibration analysis of arbitrary beams [18]. The unified formulation allows any order beam theory to be systematically analyzed by one-dimensional Finite Element Method. Complicated structures have been studied by the method in Refs. [19,20]. Beams with non-structural masses have been studied in Refs. [21,22]. High order generalized beam theories give accurate result in comparison with 3D FE-models. However, even if these analyses require significantly less computational effort than the 3D FEM, several hundred DOFs are still needed. Thus, there is a need to develop the classical simple beam models further to account the effects of local cross-section deformations on the global beam level modes.

This paper provides a method to take the inertia induced local deformation effects into account in classical beam theories. The correction is based on the energy involved in the cross section deformation. It can be used to correct the beam modal results independent of the solution of the beam problem. This allows use of detailed numerical models where necessary, while carrying out straightforward parts of the problem effectively by simple analytical formulae. This kind of approach has value in conceptual design of structures in which the accuracy of solutions must be reasonable, while the computational cost must be extremely light.

2. Method definition

2.1. Assumptions and limitations

The beam axis is denoted with x-coordinate, while the vertical direction is denoted by z-coordinate. This study is limited to analysis of

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Nomenclature

A	Amplitude, (Area in Eq. (47))
$\mathbf{b}$	Breadth
$\mathbf{e}$	Unit vector
E	Young's modulus
f	Frequency in Hz
G	Shear modulus
h	Height
I	Second moment of area
K	Generalized Stiffness
L	Length
m	Mass per unit length of beam
M	Generalized Mass
q	Distributed load
r	Response
S	Stiffener spacing
t	Time
T	Kinetic energy
U	Strain energy
w	Displacement
x, y, z, s, n	Coordinates

Greek Symbols

δ	Convergence limit
θ	Thickness
λ	Wave length
ν	Poisson's ratio
ξ	Generalized coordinate
ρ	Mass density of the material
Ψ	Deflection mode shape
ω	Frequency in rads^{-1}

Subscripts & Superscripts

*	Corrected value
B	Global beam
C	Cross section
CR	Cross section quantity Relative to global point unit amplitude enforced harmonic motion
dyn	Dynamic
ef	Effective
i	Mode number
n	Iteration step number
peak	Peak value
PU	Periodic unit
S	Stiffener
sta	Static

beams with symmetric cross sections with respect to xz-plane. Applied coordinate system, and structural dimensions are presented in Fig. 1a. Cross section coordinate s goes around centerline of the cross section, and n is perpendicular to s.

Small amplitude free vibration is assumed. Cross sections are assumed to have global reference points(s) that follow global bending behavior of the beam axis. The global reference points are defined in stiff points of the cross section, typically in intersections of plated parts. This study includes thin-walled cross sections, in which thicknesses θ are significantly smaller than cross section main dimensions ($\theta \ll b$ & $\theta \ll h$). Examples of possible cross sections and their global reference points are presented in Fig. 1b.

Length of flexural waves of beam vibration is assumed long in comparison with cross section dimensions ($b \ll \lambda$ & $h \ll \lambda$). This means that locally in a point of cross section, the transversal bending stiffness of cross section dominates the longitudinal bending stiffness by nodes of

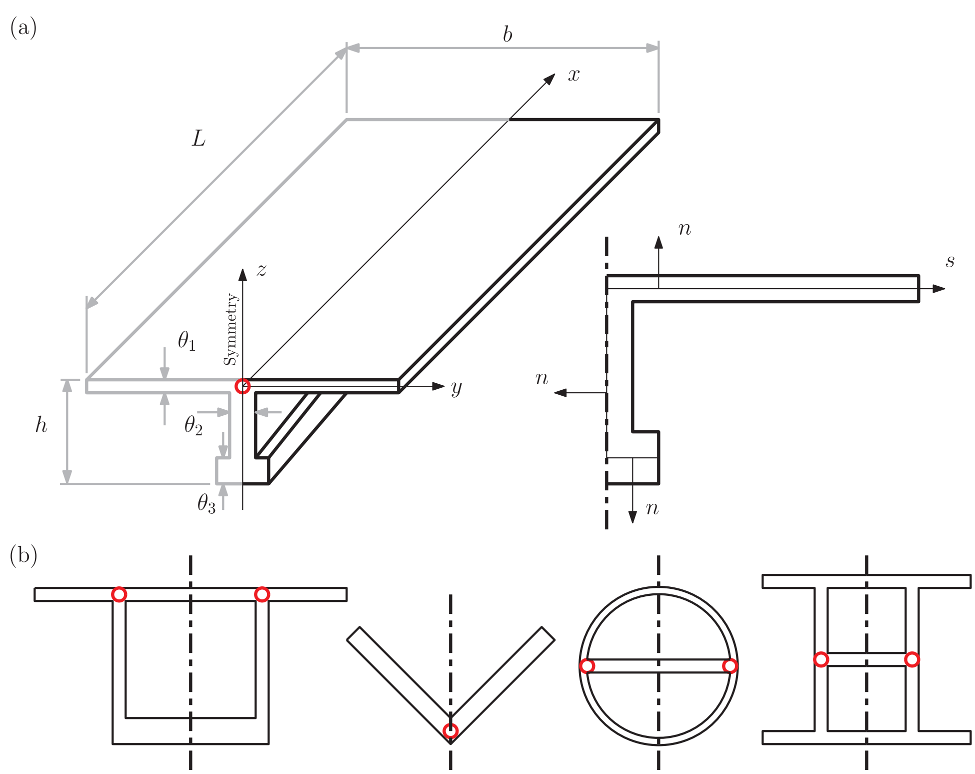

Fig. 1. (a) Definition of coordinate systems and dimensions. (b) Examples of thin-walled symmetric cross sections, global reference points indicated by circles.