Thin-Walled Structures 119 (2017) 457–469

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

An analytical and numerical study on the buckling of cracked cylindrical shells

V. Akrami[a], S. Erfani[b,*]

[a] Faculty of Engineering, University of Mohaghegh Ardabili, Ardabil, Iran
[b] Department of Civil and Environmental Engineering, Amirkabir University of Technology, Tehran, Iran

ARTICLE INFO

Keywords:
Cracked cylindrical shell
Buckling load
Beam on elastic foundation
Finite element method
Eigenvalue analysis

ABSTRACT

Presence of flaws or cracks may cause substantial decrease in the strength of a component or structure. This kind of structural damage will accumulate over time, leading to significant decrease in ultimate load carrying capacity of structural members and premature brittle failure. In this paper, a coupled analytical and numerical study is implemented in order to evaluate buckling load of cracked cylindrical shells which is often encountered in the fields of civil engineering, offshore engineering and mechanic engineering. In the first phase of the study, buckling load of cylindrical shells with circumferential crack is investigated by treating the symmetric model as a cracked rectangular beam-column on an elastic foundation. The cracked beam-column is modelled as two beam elements on elastic foundation connected by an equivalent rotational spring and governing characteristic equation is obtained. Finite element models are used to verify analytical results and assess their accuracy, based on which good agreement is observed between analytical and numerical results. Effect of different parameters such as the length of cylinder, radial stiffness of cylinder, crack location and crack severity is studied on the axial load carrying capacity of such members. A simplified equation is proposed in order to calculate buckling load of tall cylindrical shells with a circumferential crack. In the second phase, the buckling load of cylindrical shells with a partial crack is studied using the results of finite element analysis on models with varying crack length. Based on the results of numerical analysis, an empirical equation is proposed to interpolate buckling load of these members between two limiting values, namely the buckling loads of same uncracked cylinder and the cylinder with circumferential crack.

1. Introduction

The strength of a compression member may be reduced significantly by the presence of defects. The most common type of these defects is due to the presence of small initial cracks which will grow under fatigue loads. The awareness of this phenomenon in metallic structures started in the mid-19th century with the occurrence of fatigue failures in the railway industry [1]. Presence of cracks in a compressive cylindrical shell can decrease buckling load of the member and cause premature brittle failure. This phenomenon is one of the major failure modes in columns of highway and railway bridges, cranes, offshore structures, containers, machine elements, etc.

Buckling behaviour of cracked cylindrical shells has been studied by several researchers. Among the rest, Hutchinson et al. [2] conducted a joint theoretical and experimental investigation on the effects of certain types of local axisymmetric imperfections on the buckling of cylindrical shells. Considering the analogy between the strut on an elastic foundation and the axisymmetric cylinder El Naschie [3] presented a closed form solution for the post buckling path of an infinitely long cylindrical shell with a free edge. It is stated that the presented solution could relate to the buckling of a compressed concrete cylinder containing a circumferential crack. Vafai et al. [4] computed the free vibration frequencies and the corresponding mode shapes of a simply supported rectangular plate with a crack emanating from one edge. Once the vibration frequencies and mode shapes were found they were used to establish regions of instability and compute buckling loads of the plate. Hampton and Nelson [5] conducted a numerical and experimental study on the crack growth in thin plates and cylinders and showed that the buckling effects must be included in the finite element analysis to have a good agreement between numerical and experimental results. In an attempt to study more general case, Brighenti [6,7] studied rectangular elastic thin-plates a through thickness crack and assessed the effect of crack length, crack orientation and boundary conditions of the plate on the buckling loads. Most of the recent studies in this filed are focused on finite element (FE) modelling to investigate effect of different parameters (e.g. crack length, crack angle, loading condition,

* Corresponding author.
E-mail addresses: v.akrami@uma.ac.ir (V. Akrami), sderfani@aut.ac.ir (S. Erfani).

http://dx.doi.org/10.1016/j.tws.2017.06.023
Received 20 December 2016; Received in revised form 13 June 2017; Accepted 20 June 2017
0263-8231/

Nomenclature

a	depth of crack (L)
b	length of crack (L)
C_m	flexibility of rotational spring _considering the moment per unit width_ (F^{-1})
$D = Et^3/[12(1-\upsilon^2)]$	bending stiffness of the plate (FL)
E	modulus of elasticity (FL^{-2})
$f(R_k,\varphi)$	function defining the transition of k_p^2 between two boundary values, k_u^2 and k_c^2
$f(\xi)$	the local flexibility function of the cracked section
i	integer number, $i = 1, 2, 3, \ldots$
$k^2 = P_s/D$	the axial load factor of the longitudinal strip (L^{-2})
k_u^2	the axial load factor of uncracked cylinder
k_c^2	the axial load factor of cylinder with circumferential crack
k_p^2	the axial load factor of cylinder with partial crack
l	the distance between the crack and the reference edge of tubular section (L)
L	length of tubular section (L)
$m = \alpha/D$	bed stiffness factor of longitudinal strip; radial stiffness factor of cylinder (L^{-4})
M_j	moment at the end of the jth segment at the location of crack _per unit width_ (F)
n	the number of half sine waves in which the column is subdivided at buckling
N	external pressure on the tubular section (FL^{-2})
P_{cr}	buckling load of a tubular section (F)
P_s	buckling load of the longitudinal strip _per unit width_ (FL^{-1})
q_1, q_2	coefficients of the regression equation used to calculate k_c^2
r	radius of tubular section (L)
$R_k = k_c^2/k_u^2$	the ratio between the axial load factors of the circumferentially cracked and uncracked cylinders
t	thickness of tubular section (L)
V_j	shear at the end of the jth segment at the location of crack _per unit width_ (FL^{-1})
x_j	distance from the reference point on the jth part of the longitudinal strip (L)
y_j	deflection at x_j (L)
α	the stiffness of the elastic foundation (FL^{-3})
$\beta = l/L$	the crack location factor
δ_r	radial displacement of cylindrical shell under external pressure (L)
θ_j	slope of the jth segment at the location of crack
λ_1, λ_2	roots of characteristic equation
υ	Poisson's ratio
$\xi = a/t$	crack depth ratio
$\varphi = b/2\pi r$	crack length ratio
$\psi = C_m \times D$	parameter showing flexibility of cracked section

material properties and thickness of the shell) on the buckling behaviour of cracked cylindrical shells [8–11] while some others include laboratory tests for this purpose [12,13]. However, there is limited or no practical mathematical tool available in order to evaluate buckling load of cracked cylindrical shells. Also, most of the mentioned studies have considered cylindrical shells with a through the thickness crack which is not the case in most of the real situations such as the cords of offshore platforms, cylindrical containers and columns of bridges. In this paper, a novel analytical method is used to formulate buckling behaviour of circumferentially cracked cylinders with varying column length, crack location and crack depth. For this purpose, the symmetric model is treated as a cracked rectangular beam-column on an elastic foundation and its buckling load is calculated. The solution to the buckling of intact beam-column on an elastic foundation can be found in Refs. [14–16]. The most recent work in this field has been conducted by Xia and Zhang [17] who conducted a FE analysis in order to calculate critical buckling load of a beam supported on an elastic foundation. Unlike the case of ordinary cracked beam-columns, research conducted on the buckling of cracked beam-column on elastic foundation is very limited. In this regard, Wang [18] studied buckling of a beam-column on an elastic foundation with an interior hinge (with zero rotational stiffness) and determined optimum hinge location for various foundation stiffnesses. In another work, Wang [19] formulated buckling equations for an infinite beam on an elastic foundation containing one or more weakened joints. In the most recent work, Melissianos and Gantes [20] numerically modelled a beam-column with one middle or two equally spaced internal flexible joints resting on elastic foundation. Results of this analysis are used to study upheaval buckling of buried pipelines equipped with flexible joints for their protection against activation of reverse seismic faults.

In the current study, the cracked beam-column is modelled as two beam elements with finite length resting on an elastic foundation connected by an equivalent rotational spring and governing characteristic equation is obtained. Stiffness of the rotational spring is

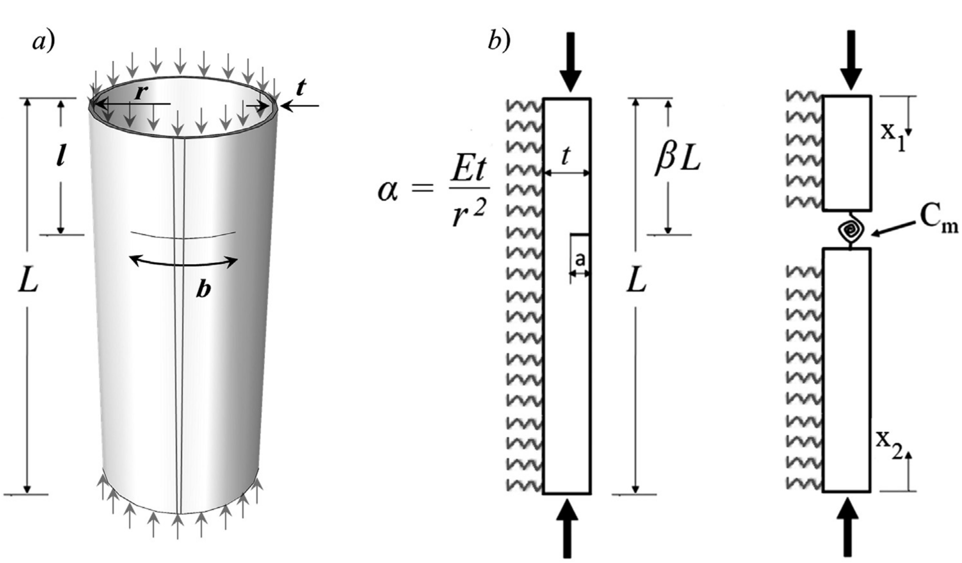

Fig. 1. Model configuration; *a*) cracked cylindrical shell; *b*) simplified model (cracked beam on elastic bed).