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Identification of buckling modes in generalized spline finite strip analysis of cold-formed steel members

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ABSTRACT

In this paper, a mode identification technique in the context of spline finite strip method (SFSM) is presented to compute the contribution of primary (global, distortional and local) and secondary (shear/transverse extension) buckling modes. The base vectors corresponding to individual buckling modes are developed based on the principles of generalized beam theory. The buckling mode shape in SFSM is approximated as a linear combination of these orthonormal base vectors to evaluate the participation of individual buckling mode. The proposed mode identification technique is able to successfully quantify the participation of different buckling modes and the mode participation is comparable with mode identification using finite strip method (FSM) and generalized beam theory (GBT). Illustrative examples are presented to calculate the participation of individual modes in cold-formed steel sections under different loading and boundary conditions. Also the specific application of mode identification in SFSM is demonstrated.

1. Introduction

Cold-formed steel design is governed mainly by buckling of the cross section or member as a whole and design of sections using Direct Strength Method (DSM) incorporates the following three primary buckling modes; local, distortional and global buckling. Generally local buckling has significant post-buckling strength reserve, distortional buckling has moderate and global buckling has negligible reserve. Hence, identification of individual buckling mode in a generalized deformation mode becomes important in the calculation of post-buckling capacity for the design of cold-formed steel members. Also the calculation of elastic local, distortional and global buckling stresses is a prerequisite for design using DSM.

Elastic buckling stresses are determined analytically or by using numerical methods. Elastic local and global (Euler) buckling stresses are derived analytically in Timoshenko and Gere [1], whereas in the case of distortional buckling, elastic buckling stress is determined from analytical model of flange-lip combination with translational and rotational stiffness [2–4]. Numerical evaluation of elastic buckling stresses are performed traditionally using finite element, finite strip or spline finite strip method. The finite strip method (FSM) being used in DSM for the calculation of buckling stresses incorporates trigonometric function in the longitudinal direction and polynomial interpolation function in the transverse direction of the plate. The continuous interpolation function in longitudinal direction makes the incorporation of intermediate support and concentrated loads difficult in FSM. The discontinuities and variations in longitudinal direction are incorporated in finite element method (FEM) and spline finite strip method (SFSM).

The buckling analysis of cold-formed steel sections using the classical methods result in numerous buckling modes and the designer's interest lies in the determination of local, distortional and global buckling modes for design using DSM. Another approach for the determination of elastic buckling stresses is generalized beam theory (GBT) which is an extension of classical beam theory transformed from nodal to modal degree of freedom. The basic principles of GBT for the analysis of cold-formed steel sections with cross sectional distortion was introduced in [5]. Explicit equations for individual buckling modes under axial load and uniform bending moment by considering geometric nonlinearity in basic equations of GBT was introduced by [6]. To decompose the FSM buckling solutions into pure global, distortional, local and shear/transverse extension modes in cold-formed steel open cross sections, GBT principles were integrated into FSM [7–9]. The method known as constrained finite strip method (cFSM) was extended to closed and branched cross sections for decomposing local buckling mode from combined global-distortional mode [10]. In the context of FEM, the GBT principles were introduced to constrain the finite element model to buckle in a particular pure buckling mode [11,12]. Constrained finite element method has been presented [13,14], which enforce the finite element model to buckle in pure local, distortional and global buckling modes. A thin-walled shell finite element was proposed

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in the context of constrained shell finite element analysis [15] to calculate pure deformation modes. In SFSM, decomposition of buckling modes into combined global-distortional and local mode was proposed [16]. In a recent study by authors [17], a constrained spline finite strip method (cSFSM) was proposed to decompose buckling modes into pure local, distortional and global buckling modes.

An approach known as mode identification technique has been used by researchers to compute the contribution of individual buckling modes from generalized buckling mode shapes. The mode identification technique using cFSM base functions has been successfully implemented in the FSM buckling mode shapes [18]. The various factors determining the calculation of mode contributions like choice of base functions, orthogonalization and normalization of vector spaces were assessed in [18]. The mode identification in FSM was also extended to members with general end boundary conditions [19]. The mode identification technique has been implemented in shell finite element analysis for cold-formed steel sections subjected to uniaxial compression and bending [20,21]. GBT cross sectional deformation modes were also introduced to evaluate the participation of local, distortional and global modes in FEM buckling mode shapes [22]. To the authors' knowledge, this is the first time mode identification study in the context of spline finite strip method is presented.

In the present study, the participation of global, distortional, local and other (shear/transverse extension) deformation modes are evaluated from generalized buckling mode shapes in SFSM. The base functions in the context of constrained spline finite strip method (cSFSM) are derived based on cFSM and GBT principles. The calculation of mode participation in SFSM is performed for cold-formed steel sections under various loading and boundary conditions. The proposed mode identification technique using SFSM is advantageous compared to FSM in incorporating discontinuous boundary and loading conditions. Also the analysis using SFSM is computationally less intensive than mode identification using traditional finite element techniques. Even though GBT studies are available for mode identification, the proposed method is yet another credible formulation for mode identification in the classical framework.

2. Base functions for mode identification in SFSM

The base functions for local, distortional, global and shear/transverse extension (other) modes are determined from mechanical assumptions of deformation modes based on GBT principles. These base functions being represented as vectors of nodal displacements, are also termed as base vectors. For calculation of mode contribution, the base vectors needs to be orthogonalized and normalized. The displacements corresponding to buckling modes in an SFSM analysis are represented as linear combination of base vectors. The SFSM formulation and the steps for determining the base vectors are briefly presented in this section.

2.1. SFSM formulation

In SFSM, the thin-walled plate element is modelled as '2D' strips having four degree of freedom along each section knot, 'u', 'v', 'w' and 'θ_{xz}' as shown in Fig. 1a. For in-plane deformations 'u' and 'v', 2D plane stress condition is assumed whereas for out of plane displacements 'w' and 'θ_{xz}', Kirchhoff's plate theory is employed. The displacement functions are expressed as the product of nodal displacements and shape functions in longitudinal and transverse direction. In transverse direction, Lagrangian interpolation function is assumed for membrane (in-plane) displacements and Hermitian interpolation function for flexural (out of plane) displacements. In the longitudinal direction, cubic spline (B3) having four sections shown in Eq. (1) is implemented (Fig. 1b). Spline amendment schemes proposed by Fan [23] by including one additional section knot at each end (Fig. 1c) satisfying natural and geometric boundary conditions has been adopted.

$$\phi_i(y) = \frac{1}{6h^3}\begin{cases}(y-y_{i-2})^3 & y_{i-2}\le y\le y_{i-1}\\ h^3+3h^2(y-y_{i-1})+3h(y-y_{i-1})^2-3(y-y_{i-1})^3 & y_{i-1}\le y\le y_i\\ h^3+3h^2(y_{i+1}-y)+3h(y_{i+1}-y)^2-3(y_{i+1}-y)^3 & y_i\le y\le y_{i+1}\\ (y_{i+2}-y)^3 & y_{i+1}\le y\le y_{i+2}\\ 0 & otherwise\end{cases} \tag{1}$$

The strain-displacement relation and stress-strain relation are established and by applying the principle of minimum total potential energy, the stiffness equation is developed. The membrane and flexural characteristics has been combined for analysing folded plate member like cold-formed steel section. The transformation matrix is used to transform the element stiffness equation from local to global direction and stiffness matrices are assembled to form global stiffness matrix. For buckling analysis, the increase in potential energy of membrane forces due to flexural and membrane deformations, developed by Plank and Wittrick [24] has been adopted. The generalized Eigen buckling equation has been formulated by applying the principle of minimum total potential energy as shown in Eq. (2), where $[K]$ and $[K_g]$ are the elastic and geometric stiffness matrices respectively, 'λ' is the buckling load factor and $[\Delta]$ is the matrix of Eigen vectors corresponding to various buckling modes.

$$([K]-\lambda[K_g])[\Delta]=0 \tag{2}$$

2.2. Base vectors for buckling modes

The base vectors corresponding to buckling modes are developed from mechanical criteria based on GBT principles. The displacement field used for the determination of base vectors are identical to SFSM. The restraint matrices, 'R_G', 'R_D', 'R_L' and 'R_O' corresponding to local, distortional, global and other (shear/transverse extension) deformation modes are developed based on these criteria. The criteria considering the co-ordinate system shown in Fig. 1a are illustrated below:

(i) Transverse extension, 'ε_x' and in-plane shear strain, 'γ_{xy}' of a flat plate strip has to be zero. Also the local displacement 'v' has to be linear in 'x' direction within the flat strip.
(ii) The cross section elements are associated with non-zero warping displacements. Also the cross section is in transverse equilibrium.
(iii) There is no transverse flexure of the cross section, ie the cross section is undistorted.

For global buckling (G), all the three criteria has to be satisfied whereas in the case of distortional buckling (D), criteria (iii) is violated. By applying the mechanical criteria, the restraint matrix for global-distortional subspace, 'R_{GD}' is developed by relating the warping displacements at edges of flat plate with other degree of freedom. The 'R_{GD}' matrix is decomposed into 'R_G' and 'R_D' matrix by multiplying with warping displacements corresponding to global and distortional modes derived from GBT cross sectional analysis. In the case of local buckling (L), both criteria (ii) and (iii) are violated and the possible displacements are local displacement in local 'z' direction of sub nodes and external nodes and rotation 'θ_{xz}' of all nodes. The base vectors for 'R_L' matrix are generated by imposing unit displacement at possible degree of freedom of each node keeping all other degree of freedom as zero. The restraint matrix for shear/transverse extension (other) modes (O), 'R_O' includes all the deformation modes which are not included in global, distortional or local category and hence all the mechanical criteria are violated here. The restraint matrix, 'R_O' is determined from the null space of 'R_{GDL}' subspace which is a combination of global, distortional and local sub space as shown in Eq. (3).

$$R_o^T R_{GDL}=0 \tag{3}$$

The restraint matrices developed for individual modes consists of linearly independent base vectors of nodal displacements of cross