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An analytic method for free and forced vibration analysis of stepped conical shells with arbitrary boundary conditions

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ABSTRACT

An analytic method is presented for free and forced vibration analysis of stepped conical shells with general boundary conditions. The method is involved in dividing the stepped shells into segments according to the locations of discontinuities, such as thickness and semi-vertex angle. Combining Flügge shell theory with power series method, displacements and forces at cross-section of conical segments are expressed in terms of eight unknown coefficients. Meanwhile, by employing artificial springs to restrain edge displacements, arbitrary boundary conditions, including classic and elastic ones, can be analyzed. All separate segments are assembled together through displacement continuity conditions and equilibriums of forces at junctions of discontinuities. To test the validity of present method, comparisons of some stepped conical shells subjected to classic and elastic boundary conditions are firstly presented. The results of present method agree fairly well with those in literature and calculated by finite element method. Furthermore, influences of semi-vertex angle, elastic boundary conditions, discontinuity, excitation and damping are investigated. Parametric studies reveal that meridional and circumferential displacements have significant effects on fundamental and beam mode frequency parameters, and effects of the location of discontinuity and semi-vertex angle depend on boundary conditions and thickness ratios.

1. Introduction

Conical shells play a very significant role in many engineering applications, such as aircrafts, submarines, loudspeakers and so forth, and vibration analysis of conical shells is important for the safety and stability. Correspondently, dynamic characteristics of conical shells with uniform thickness have been widely investigated through the Rayleigh-Ritz method [1–4], transfer matrix method [5], power series method [6], generalized differential quadrature method [7], dynamic stiffness method [8], Galerkin's method [9,10] and finite element method [11] and others. To accommodate more complex requirements in practical engineering applications, thickness of conical shells is usually designed to vary with meridional location. In general, the variation of thickness can be classified into two forms, namely continuous variable thickness and stepped thickness. In contrast with stepped thickness, vibrations of conical shells with continuous variable thickness have been studied by lots of scholars [12–21]. However, stepped thickness is more common in practice. Furthermore, stepped semi-vertex angle is also another common form of stepped conical shell. On the whole, the research about stepped conical shells, including thickness and semi-vertex angle, is much rare, which may be attributed to the complexity involved in modeling and solution process. Of course, finite element method (FEM) is a powerful tool to overcome those difficulties. Meanwhile, commercial programs of FEM, e.g. ANSYS, ABAQUS, NASTRAN and so on, have been well developed, which can expand the application scopes of FEM in some degree. Nevertheless, efficiency is the fatal drawback of FEM. For instance, scholars and engineers have to classify solutions one by one to identify the mode shape of a particular natural frequency, which is a time-consuming, error-prone and tiresome progress. Furthermore, the number of elements rapidly increases as the analysis frequency increases, which significantly increases computing time and storage space. In consequence, proposing a unified approach to accurately and efficiently analyze free and forced vibrations of stepped conical shells with arbitrary boundary conditions is worth considering.

Harintho and Logan [22] used the flexibility method to identify discontinuity stresses of conical shells consisting of segments of different thicknesses. Zhou and Lei [23] presented an asymptotic transfer function method for analysis of conical shells with stepped conical angle or thickness. By expanding displacement functions as Fourier series in circumferential direction, motion equations were decoupled into a group of partial differential equations with one space

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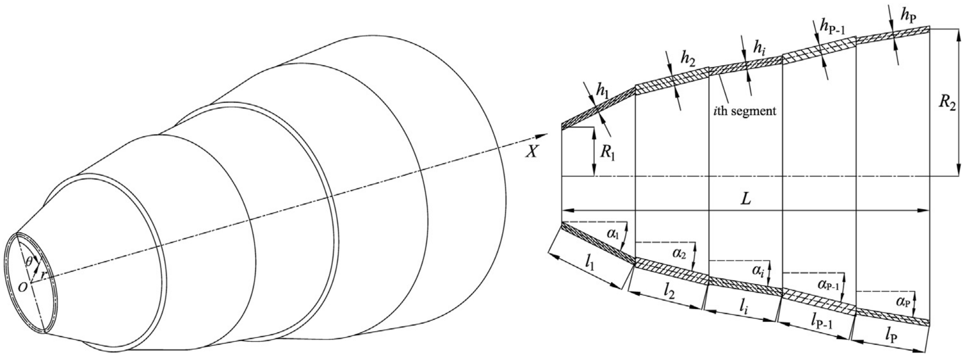

Fig. 1. Schematic diagram of a P-stepped conical shell.

variable and one time variable. In addition, introducing a small perturbation parameter and using the Laplace transformation and perturbation technique, the partial differential equations with variable coefficients were reduced to ordinary differential equations with constant coefficients, which could be solved by the transfer function method. Qu et al. [24] proposed a domain decomposition method to study free and forced vibrations of conical shells with stepped thickness. Fourier series and Chebyshev orthogonal polynomials were adopted as admissible functions for every shell domain with uniform thickness. A modified variatonal principle and the least-squares weighted residual method were used to satisfy boundary and continuity conditions. To accommodate the computing requirement of high-order vibration modes and responses, segments with uniform thickness were divided into narrower strips. Based on Donnell shell theory, Shakouri and Kouchakzadeh [25] studied free vibrations of joined conical shells, which was the general form of stepped conical shells. The displacement functions were expanded as trigonometric functions in circumferential direction and power series in meridional direction. Then, segments were assembled through continuity conditions.

As the special cases of stepped conical shells, dynamic characteristics of combined conical-cylindrical shells have been studied by some but not many scholars. Hu and Raney [26] used the multisegmental numerical integration technique to study free vibrations of jointed conical-cylindrical shells, and the results were validated by experiments. Irie et al. [27] presented a transfer matrix method for free vibrations of joined conical-cylindrical shells. Patel et al. [28] used the finite element method to analyze free vibrations of laminated anisotropic composite conical-cylindrical structures. Efraim and Eisenberger [29] used the dynamic stiffness matrix method to determine exact vibration frequencies of segmented axisymmetric shells, and natural frequencies and modes of a conical-cylindrical shell were presented. Caresta and Kessissoglou [30] used the power series method and wave propagation method to study free vibrational characteristics of isotropic coupled cylindrical-conical shells. As similar with the method in Ref. [24], a variational method was proposed by Qu et al. [31] to study free vibrations of joined cylindrical-conical shells with classic and elastic boundary conditions. Ma et al. [32] presented a modified Fourier-Ritz method for free and forced vibrations of coupled conical-cylindrical shells with arbitrary boundary conditions. The displacement functions of conical and cylindrical shells were expanded as a modified Fourier series consisting of standard Fourier series and closed-form supplementary functions. Artificial springs were used to combine adjacent segments.

In above cited literature about stepped conical shells, the literature investigating both free and forced vibrations of conical shells with stepped thickness and semi-vertex angle is much rare. In addition, classic boundary conditions, such as free, clamped and simply supported ones, were emphatically studied since the displacements can be easily expanded as special kinds of functions, e.g. trigonometric functions or their combinations. However, boundary conditions may be not classic ones in practice engineering. To this end, the main purpose of this paper is to present a unified approach to determine natural frequencies and forced vibration responses of stepped conical shells with arbitrary boundary conditions. The approach is involved in dividing the stepped shells into narrow segments at the locations of discontinuities of thickness and semi-vertex angle. Flügge theory is used to describe equations of motions of conical segments and corresponding displacement functions are expanded as power series. In order to deal with arbitrary boundary conditions, artificial springs are utilized to restrain displacements at two edges. After all separate segments analyzed, they are assembled by using continuity conditions of adjacent segments. Comparisons of free and forced vibration results of present paper and those in literature and calculated by FEM demonstrate high accuracy and wide application of present method. Furthermore, present method is believed to include following novelties. First, it offers a unified method to analyze both free and forced vibration characteristics of stepped conical shells, the literature about which is rare. Second, both classic and elastic boundary conditions can be dealt with. Last but the most important, in contrast with FEM, present method has much higher efficiency while high accuracy can be guaranteed.

2. Theory equations

2.1. Method description

Fig. 1 shows schematic diagram of a stepped conical shell consisting of P segments. (r, θ, X) is the global cylindrical coordinate system, the origin of which is located at the center point of left end plane. h_i, l_i and α_i are the thickness, meridional length and semi-vertex angle of the ith segment. R_1 and R_2 are the radii of two ends, and L is the total axial length. Due to the discontinuities, the stepped shell is firstly divided into narrow segments with uniform thickness and semi-vertex angle. For forced vibration analysis, if the external excitation is not forced at one junction of discontinuity, the segment with excitation needs to be divided again at the location of excitation. Regardless of boundary conditions, segments are analyzed individually. By using continuity conditions of adjacent segments, all segments can be easily assembled to the stepped shell and corresponding governing equation can be established with the help of boundary conditions at two ends.

2.2. Motion equations of segments

Fig. 2 shows displacement and force resultants of a conical segment with semi-vertex α. x is the meridional coordinate and it is measured from the middle of the segment. Circumferential coordinate θ is the same with the one of global coordinate system. u, v and w are orthotropic displacements in meridional, circumferential and normal directions, and $\beta = \partial w / \partial x$ is slope. N, $\overline{T}$, $\overline{S}$ and M are force resultants at cross-section of conical shells and their expressions are given in Appendix A.