Thin-Walled Structures 116 (2017) 124–135

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Free vibration analysis of isogeometric curvilinearly stiffened shells

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ARTICLE INFO

Keywords:
IGA
Curvilinearly stiffened shells
free vibration

ABSTRACT

The isogeometric analysis (IGA) proposed by Hughes is a new approach in which Non-Uniform Rational B-Splines (NURBS) are used as a geometric representation of an object. It has superiorities of capturing exact geometry, simplifying refinement strategy, easily achieving degree elevation with an arbitrary continuity of basic functions and getting higher calculation accuracy. In this paper, the IGA approach is extended to solve the free vibration problem of curvilinearly stiffened cylindrical and shallow shells. The first-order shear deformation theory (FSDT) and the Reissner-Mindlin shell theory are used to model the shells, and the three-dimensional curved beam theory is employed to model the stiffener which can be placed anywhere within the shell. Some numerical examples are solved to study the vibration behavior of the curvilinearly stiffened shells. The effects of shell and stiffener element numbers, boundary conditions, stiffener ply modes and shell thicknesses on the natural frequency are investigated. Results have shown the correctness and superiorities of the present method by comparing the results with those from commercial finite element software and some numerical methods in existing literatures. One of the advantages is that the element number is much less than commercial finite element software, whereas another is that the mesh refinement process is much more convenient compared with traditional finite element method (FEM).

1. Introduction

Stiffened shells have been widely used in aircraft fuselages, missile bodies, submarines and roofs for decades. These structures can achieve better stiffness and strength than normal shells as well as save the structure material and reduce the weight, which improve the utilization efficiency and economy.

During the last several decades, an increasing number of researches on stiffened shells have been conducted. Earlier studies on stiffened cylindrical shells using the finite element method (FEM) can be found in the paper of Hoppmann [1]. Orthogonal plate model was used to study the vibration of stiffened cylindrical shells where the stiffeners laid vertically. After that, Stanley and Ganesan [2] studied the free vibration of clamped stiffened cylindrical shells, in which both short and long shells were discussed, and the effects of stiffener type, number and laid form on the natural frequency were investigated. Additionally, Nayak and Bandyopadhyay [3] used the FEM to study the free vibration of several forms of stiffened shallow shells, including ellipsoidal, hyperboloid and conical shells. In order to study the free vibration of stiffened shell, Samanta and Mukhopadhyay [4] developed a three-node triangular shell element combining Kirchhoff triangular plate bending element with Allman plane stress element. Pan et al. [5] studied the free vibration of stiffened cylindrical shells under arbitrary boundary conditions. Efimtsov and Lazarev [6] researched the forced vibration of stiffened plates and cylindrical shells. Recently, Balamurugan and Narayanan [7] studied the free vibration of stiffened piezoelectric plates and shells.

For the curvilinearly stiffened plates and shells, there already has been a number of work being completed based on FEM as well. Shi et al. [8,9] presented some researches on curvilinearly stiffened plates and shells using FEM, e.g. (1) the static, vibration and buckling analysis of curvilinearly stiffened plates; (2) the vibration with in-plane loading of curvilinearly stiffened plates; (3) the free vibration analysis of curvilinearly stiffened cylindrical shells. To study the stability behavior of complex shaped and multi-functional structure with the concept of integrated and bonded unitized structural components, Zhao and Kapania [10] proposed an efficient finite element buckling analysis of unitized stiffened composite panel stiffened by arbitrarily shaped stiffeners.

In addition to the FEM, a variety of other numerical methods have been widely applied to dynamic stiffened plates and shells. Li [11] gave a perturbed solution for the free vibration of ring-stiffened cylindrical shells. Cheng and Dade [12] used Gaussian spline collocation method to study the dynamic behavior of stiffened shells and plates. Mustafa and Ali [13] calculated the natural frequency of stiffened cylindrical shells using Ritz Method. Shi et al. [14] used Ritz Method to solve the

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http://dx.doi.org/10.1016/j.tws.2017.03.019
Received 1 September 2016; Received in revised form 14 December 2016; Accepted 15 March 2017
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vibration problem of curvilinearly stiffened shallow shells. Jafaria and Bagheri [15] studied the free vibration of ring-stiffened cylindrical shell. In their work, several methods were used, such as FEM, Ritz method and experimental approach. They discussed the influence of non-uniform rib section and non-equidistant rib laying problems. Qu et al. [16] analyzed the dynamic behavior of conical-cylindrical shells blessed with ring-stiffeners using a modified variational method in which a variety of boundary conditions were considered and the displacement trial functions were combined by Fourier polynomials and Chebyshev polynomials.

In this paper, the isogeometric analysis (IGA) method proposed by Hughes et al. [17] is applied for the free vibration of curvilinearly stiffened shells, which has not been presented in the literature to the authors' knowledge. The IGA has several advantages over standard FEM, e.g. the smoothness with arbitrary continuity order, exact representation of shapes even at the coarsest level of discretization, simple and systematic refinement strategy, and more accurate modeling of complex geometries can be easily obtained. In recent years, IGA has been used in many areas such as turbulence [18–20], fluid–structure interaction [21–23], incompressibility [24–26], structural analysis [27,28], shells [29] and phase-field analysis [30]. For structural mechanics, isogeometric analysis has been extensively studied for nearly incompressible linear and non-linear elasticity and plasticity problem [26], structural vibrations [28], the composite Reissner-Mindlin plates [31], Kirchhoff-Love shells [32–34], the large deformation with rotation-free [35] and structural shape optimization [36].

The first-order shear deformation theory (FSDT) and the Reissner-Mindlin shell theory are used to model the shells in this paper, and the three-dimensional curved beam theory is employed to model the stiffener. The free vibration behavior of curvilinearly stiffened shells is studied. The stiffeners can be placed anywhere within the shell. This paper is organized as follows: In Section 2, a brief introduction of the B-spline and NURBS basis functions is considered. After that, formulations of isogeometric analysis method are presented. In Section 3, the model of curvilinearly stiffened shells is set up. Then, free vibration analysis of the curvilinearly stiffened shells is carried out. Section 4 is devoted to numerical tests which show the performance of the proposed method. In Section 5, we close this paper with some conclusions. The code is written in the FORTRAN 90. Present results are compared with the results available and those obtained using the NASTRAN software.

2. Isogeometric analysis method

2.1. NURBS basis functions

Given a knot vector which is a sequence in a non-decreasing order of parameter values, written as $\{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$ $(\xi_i \le \xi_{i+1}, i = 1, \ldots, n+p)$ where ξ_i is the i-th knot, n is the number of basis functions and p is the polynomial order. The associated B-spline basis functions for a given degree p, are defined recursively over the parametric domain by the knot vector. For $p = 0$ [17],

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

For $p \ge 1$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{2}$$

The derivatives of B-spline basis functions can be got from lower order derivatives recursively. The first order derivative of a B-spline basis function is given by

$$\frac{d}{d\xi} N_{i,p}(\xi) = \frac{p}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) - \frac{p}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{3}$$

Derivative of the k-th order is

$$\frac{d^k}{d\xi^k} N_{i,p}(\xi) = \frac{p}{\xi_{i+p} - \xi_i}\left(\frac{d^{k-1}}{d\xi^{k-1}} N_{i,p-1}(\xi)\right) - \frac{p}{\xi_{i+p+1} - \xi_{i+1}}\left(\frac{d^{k-1}}{d\xi^{k-1}} N_{i+1,p-1}(\xi)\right) \tag{4}$$

B-splines are convenient for free-form modeling. However, it cannot exactly design some common geometries in engineering. To solve this problem one usually use NURBS which are the rational functions of B-splines.

NURBS basis functions are defined as [17].

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi)\omega_i}{\sum_{j=1}^{n} N_{j,p}(\xi)\omega_j} = \frac{N_{i,p}(\xi)\omega_i}{W(\xi)} \tag{5}$$

where $N_{i,p}(\xi)$ denotes the i-th B-spline basis function of order p and ω_i is the corresponding weight. For the special case in which $\omega_i = c$, $\forall\, i$, the NURBS basis reduces to the B-spline basis.

The first order derivative of a NURBS basis function is computed using the quotient rule. That is,

$$\frac{d}{d\xi} R_{i,p}(\xi) = \omega_i \frac{N_{i,p}'(\xi)W(\xi) - N_{i,p}(\xi)W'(\xi)}{W(\xi)^2} \tag{6}$$

where

$$W'(\xi) = \sum_{i=1}^{n} N_{i,p}'(\xi)\omega_i, \quad N_{i,p}' = \frac{dN_{i,p}}{d\xi} \tag{7}$$

2.2. Formulations for IGA

Take a two-dimensional element as an example to formulate isogeometric analysis method [37]. Referring to Fig. 1 for illustration, the mapping from the parametric domain to the physical domain is given by

$$\mathbf{x} = \sum_{I=1}^{n} R_I(\boldsymbol{\xi})\mathbf{P}_I \tag{8}$$

where $\mathbf{x} = (x(\boldsymbol{\xi}), y(\boldsymbol{\xi}))$, $\boldsymbol{\xi} = (\xi, \eta)$, n is the number of the control points, R_I is the I-th shape function, $\mathbf{P}_I$ is the I-th control point.

The Jacobian matrix of the geometry mapping is defined as

$$\mathbf{J}_{\boldsymbol{\xi}} = \begin{bmatrix} x_{,\xi} & x_{,\eta} \\ y_{,\xi} & y_{,\eta} \end{bmatrix} \tag{9}$$

with the components calculated as

$$x_{,\xi} = \sum_{i=1}^{n} R_{i,\xi} x_i, \quad x_{,\eta} = \sum_{i=1}^{n} R_{i,\eta} x_i$$
$$y_{,\xi} = \sum_{i=1}^{n} R_{i,\xi} y_i, \quad y_{,\eta} = \sum_{i=1}^{n} R_{i,\eta} y_i \tag{10}$$

where $R_{i,\xi}$ and $R_{i,\eta}$ are the derivatives of the i-th shape function R_i with respect to the parametric coordinates ξ and η respectively, x_i and y_i are the coordinates of the i-th control point respectively. The determinant of $\mathbf{J}_{\boldsymbol{\xi}}$ is denoted by $|\mathbf{J}_\xi|$.

The transformation from parent domain to a parametric domain $[\xi_i, \xi_{i+1}] \times [\eta_j, \eta_{j+1}]$ is given by

$$\xi = \tfrac{1}{2}[(\xi_{i+1} - \xi_i)\bar{\xi} + (\xi_{i+1} + \xi_i)]$$
$$\eta = \tfrac{1}{2}[(\eta_{i+1} - \eta_i)\bar{\eta} + (\eta_{i+1} + \eta_i)] \tag{11}$$

where $\bar{\xi}$ and $\bar{\eta}$ represent the coordinates of Gauss point. Therefore, the determinant of the Jacobian of this transformation is

$$|\boldsymbol{J}_{\bar{\xi}}| = \frac{1}{4}(\xi_{i+1} - \xi_i)(\eta_{i+1} - \eta_i) \tag{12}$$

So, integrals of a function f with two variations x and y over the physical domain can be computed as

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