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# Stability and vibrations of a metal seven-layer rectangular plate with trapezoidal corrugated cores

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ABSTRACT

The paper is devoted to mathematical modelling of multi-layered structures. The case of a metal seven-layer rectangular plate is considered. The plate is composed of a trapezoidal corrugated main core, two inner flat sheets, two trapezoidal corrugated cores of the faces and two outer flat sheets. The direction of the corrugation of the main core is orthogonal to the one of the face cores corrugations. The hypothesis of deformation of normal to middle surface of the plate after bending and the field of displacements and stresses is formulated. The plate is simply supported and subjected to a uniformly distributed compressive loads applied to the edges. Equations of motion are derived based on the Hamilton's principle. These equations are analytically solved for static or pulsating loads. The influence of the trapezoidal corrugation pitch of the cores on the critical loads and natural frequency of the plates is analysed. The results obtained form the analytical solution are compared with the results given by the proposed finite element model.

## 1. Introduction

The sandwich structures are extensively studied since the mid of 20th century due to their many advantages. Noor et al. [1] and Vinson [2] presented a review of the problems related to computational models and analysis of sandwich structures. Carrera [3] presented the historical review of Zig-Zag theories for multi-layer plates and shells. Doyle [4] described in details the problems of stability and vibrations of structures. Lin and Zhang [5] studied the buckling and post-buckling problems of shear-deformable simply supported rectangular plates of composite construction. Thompson and Loughlan [6] presented the problem of the control of the post-buckling behaviour of various laminated plate structures. Tian and Lu [7] described the problem related to the design of compression corrugated panels. Kolakowski and Krolak [8] presented the modal coupled instabilities of thin-walled composite structures. Krolak et al. [9] experimentally studied the stability and load carrying capacity of untypical thin-walled multi-cell columns. Kotelko et al. [10] estimated the load-carrying capacity of multi-layered plate structures. Pandit et al. [11] presented an improved higher order zig-zag theory and studied the buckling of laminated sandwich plates. Domagalski and Jędrysiak [12] presented the problem of moderately large deflection of the thin periodic plates. Chen and Qiao [13] solved the shear buckling problem of composite laminated plates with the use of the Galerkin method. Magnucka and Magnucki [14] analysed the transverse shear modulus of elasticity for thin-walled corrugated cores of sandwich beams. Lewinski et al. [15] determined the transverse shear modulus of elasticity for thin-walled trapezoidal corrugated cores of seven-layer sandwich plates. Magnucka-Blandzi et al. [16] presented the shearing effect for sandwich beams with sinusoidal corrugated cores. Magnucki et al. [17] presented the bending and buckling problem of a steel composite beam with corrugated main core and sandwich faces. Malinowski et al. [18] numerically studied (with the use of FEM) the buckling and post-buckling behaviour of elastic seven-layered cylindrical shells. Lee and Fan [19] numerically analysed the bending and vibration problems of composite sandwich plates. Benjeddou et al. [20] analytically solved the free vibration problem of simply-supported piezoelectric adaptive plates basing on an exact sandwich formulation. Numayr et al. [21] analytically solved the free vibration problem of composite plates using the finite difference method. Zhang et al. [22] analysed the free vibration problem of rectangular composite laminates using a layerwise cubic B-spline finite strip method. Zingoni [23–25] presented the original theoretical models devoted to the solution of the free vibration problems of structures. Jędrysiak [26] analysed the problem of dynamic behaviour of special microstructured thin functionally graded plates. Azarboni et al. [27] investigated the nonlinear dynamic buckling of imperfect

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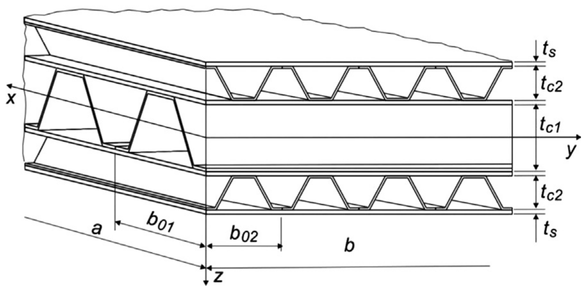


**Fig. 1.** Scheme of the seven-layer rectangular plate.

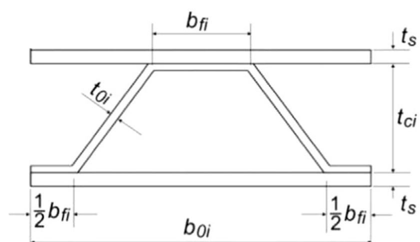


**Fig. 2.** Scheme of trapezoidal corrugations.

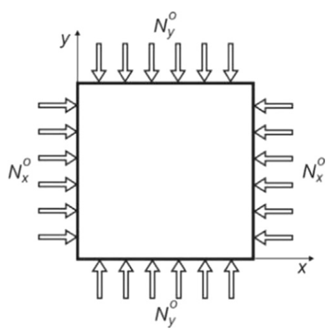


**Fig. 3.** Scheme of the in-plane compression load of the plate.

rectangular plates subjected to various pulse functions with six different boundary conditions. Stojanović et al. [28] analytically solved the vibrations and stability problems of multiple rectangular plates coupled with elastic layers based on various plate theories. Smyczynski and Magnucka-Blandzi [29] analysed the static and dynamic problems of stability of an axially compressed five-layer sandwich beam. Magnucka-Blandzi et al. [30] presented the simple theoretical model of the seven-layer plate and preliminary analysis of elastic buckling problem for static load.

The subject of the paper is the metal seven-layer rectangular plate. The plate is composed of a trapezoidal corrugated main core, two inner flat sheets, two trapezoidal corrugated cores of the faces and two outer flat sheets. The goal of the investigation is to derive the formulae for the critical load and natural frequencies. The results obtained with these formulae are to be compared with the results given by the finite element method.

## 2. Theoretical model of the seven-layer plate – strains and stresses

The seven-layer rectangular plate is shown in Fig. 1. The plate is simply supported on four edges.

The direction of the corrugations of the facings cores is orthogonal to the one of the main core corrugation. Trapezoidal corrugations of the main core and facings cores are shown in Fig. 2. Index $i = 1$ refers to the main core, while the index $i = 2$ refers to the face cores. The total depth of the cores is $t_{ci}$ and the length of one pitch of the corrugation is $b_{0i}$.

The plate is under uniformly distributed in-plane compression load with intensities $N_x^o$ or $N_y^o$ (Fig. 3).

Taking into account the layered structure of the plate, the following hypothesis of the broken line is assumed (Fig. 4), different for $x$ and $y$ direction. The straight line, normal to the middle plane of the plate before bending, does not remain straight and normal after bending. The problem of the hypothesis for multi-layer structures is described by Carrera [3] and Magnucka-Blandzi et al. [30].

Displacements, based of the assumed hypothesis, are as follows:

1) the main corrugated core for $-\frac{1}{2} \le \zeta \le \frac{1}{2}$

$$u(x, y, z) = -t_{c1}\zeta\left[\frac{\partial w}{\partial x} - 2\psi(x, y)\right], \quad v(x, y, z) = -t_{c1}\zeta\frac{\partial w}{\partial y}, \tag{1}$$

2) the inner flat sheets

- the upper sheet for $-(\frac{1}{2} + x_1) \le \zeta \le -\frac{1}{2}$

$$u(x, y, z) = -t_{c1}\left[\zeta\frac{\partial w}{\partial x} + \psi(x, y)\right], \quad v(x, y, z) = -t_{c1}\zeta\frac{\partial w}{\partial y}, \tag{2}$$

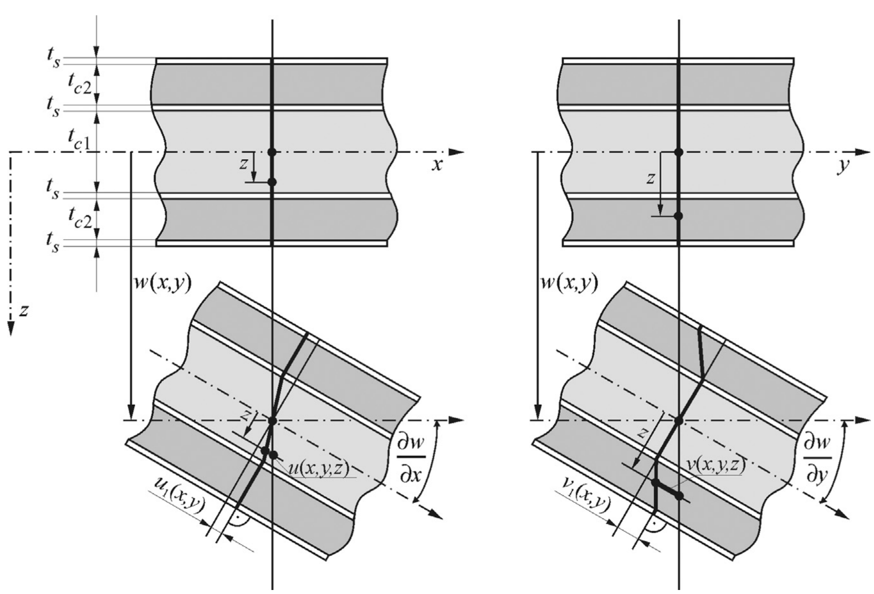


**Fig. 4.** The deformation of the normal to the middle plane of the plate.