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# Requirements for intermediate ring stiffeners placed below the ideal location on discretely supported shells

Özer Zeybek[a], Cem Topkaya[a,*], J. Michael Rotter[b]

[a] Department of Civil Engineering, Middle East Technical University, Ankara 06531, Turkey
[b] Institute for Infrastructure and Environment, University of Edinburgh, EH9 3JL Scotland, UK

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## ABSTRACT

Silos in the form of a cylindrical metal shell are commonly supported by a few discrete columns to permit the contained materials to be directly discharged. The discrete supports produce a circumferential non-uniformity in the axial membrane stresses in the silo shell. A combination of a ring beam and an intermediate ring stiffener can be used for large silos to redistribute the stresses from the local support into uniform stresses in the shell. Previous work done by the authors has identified the ideal location and the stiffness and strength requirements for the intermediate ring stiffener placed at this location. In cases where a shell with a large radius rests on a few supports, the ideal location can be quite high and the option of placing the intermediate ring stiffener below the ideal location may provide a viable solution. This paper explores strength and stiffness requirements for intermediate ring stiffeners placed below the ideal location. Pursuant to this goal, the cylindrical shell below the intermediate ring stiffener is analyzed using the membrane theory of shells. The reactions produced by the stiffener on the shell are identified. Furthermore, the displacements imposed by the shell on the intermediate ring stiffener are obtained. These force and displacement boundary conditions are then applied to the intermediate ring stiffener to derive closed form expressions for the variation of the stress resultants around the circumference to obtain a strength design criterion for the stiffener. A stiffness criterion in the form of a simple algebraic expression is then developed by considering the ratio of the circumferential stiffness of the cylindrical shell to that of the intermediate ring stiffener. These analytical studies are then compared with complementary finite element analyses that are used to identify a suitable value for the stiffness ratio for ring stiffeners placed at different locations.

## 1. Introduction

Silos in the form of cylindrical metal shells can be supported either on the ground or on a few column supports, depending on the requirements of the discharge system. If the stored granular solids are discharged by gravity, a hopper is needed at the base of the cylindrical shell with an access space beneath it to permit discharge into transportation systems. Columns at equal circumferential intervals are invariably used to elevate the silo structure and to provide the necessary access space (Fig. 1).

Depending on the size of the structure, several different support arrangements [1] may be chosen, as shown in Fig. 1. For small silos, terminating columns with rings (Fig. 1a), engaged columns (Fig. 1b) or bracket supports (Fig. 1e) may be suitable. On the other hand, medium and large silos require either columns extending to the eaves (vertical stiffeners) (Fig. 1c) or heavy ring beams (Fig. 1d) or double rings (Fig. 1f). A much greater design effort is required for the design of medium and large silos, because many more aspects become critical to the strength, as the stored weight increases as the cube of the linear dimension. This is one reason why the European Standard for Silos (EN 1993-4-1, 2007) [2] separates silo structures into different reliability classes on the basis of the overall size, with different design requirements for each class.

The design of the cylindrical shell is governed by considerations of buckling under axial compression. For cases where the axial stresses are circumferentially uniform, the classical design treatments [3–6] can be adopted where the criterion for buckling under axial compression is that for uniform compression. The presence of discrete supports, however, results in a circumferential nonuniformity of axial stresses which must be considered for buckling assessment. Previous studies of discretely supported cylinders [7–16] and those on ring beams above columns [17,18] have shown the great complexity of the behaviour.

For the most demanding situation of the large silo, a sizeable ring beam is normally used, resting on discrete supports beneath the

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[*] Corresponding author.
*E-mail address:* ctopkaya@metu.edu.tr (C. Topkaya).

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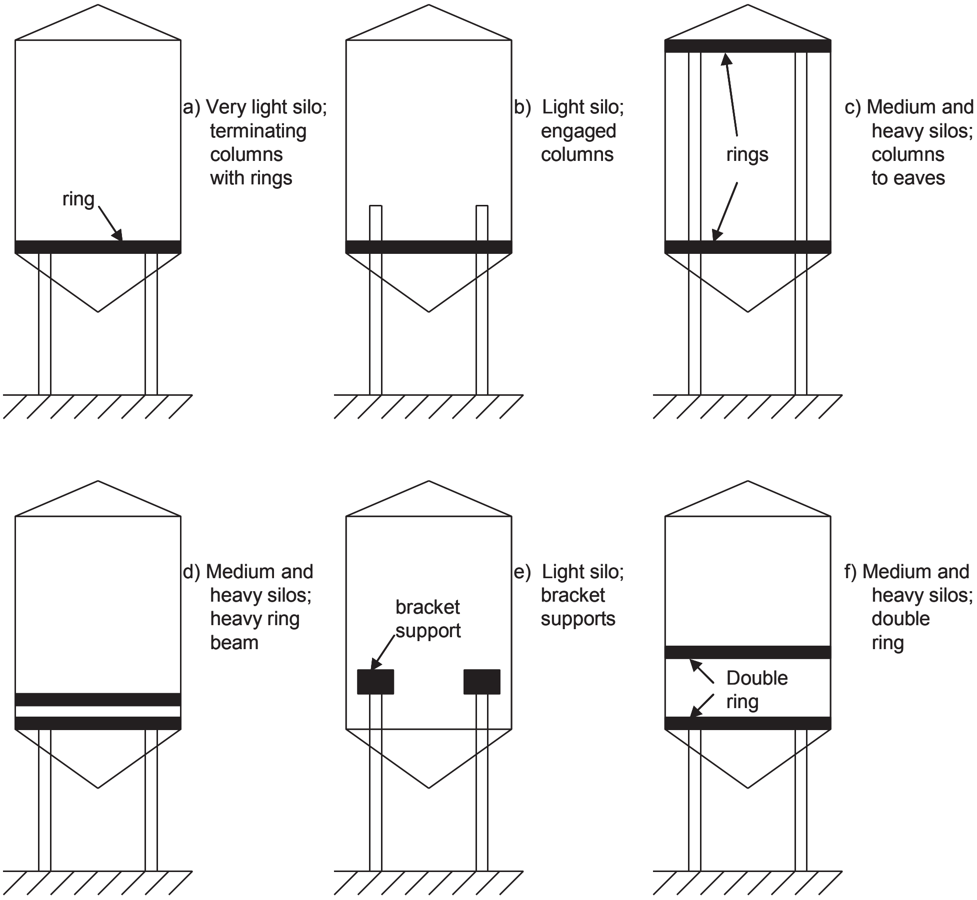

**Fig. 1.** Alternative support arrangements for discretely supported silos (after [1]).

cylindrical shell, as shown in Fig. 1d. The function of the ring beam is twofold. First, the ring beam is required to carry circumferential forces to maintain equilibrium at the transition junction between the cylinder and hopper [1]. Second, the ring beam plays an important role in redistributing the majority of the discrete forces from the column supports into a more uniform stress state in the cylindrical wall [1]. The classical design treatments [3–6] can only be valid if the ring beam properly fulfils its critical function in redistributing the discrete support loads into a relatively uniform state of stress. The extent to which this redistribution of the support forces can be achieved is directly related to the stiffness of the ring beam relative to the stiffness of the cylindrical shell (Fig. 2). Since the cylindrical shell is very stiff in its own plane, the ring beam that is subject to flexure and twisting must be remarkably stiff to be stiffer than the shell [1].

An approximate criterion to determine the appropriate ring beam stiffness was first identified by Rotter [18] and was further developed and verified by Topkaya and Rotter [19]. The criterion is based on the ring beam stiffness ratio $\psi$, which is the relative vertical stiffness of the shell to the ring beam at the base of the shell. For ring beams resting on discrete supports the peak value of axial stress occurs in the shell above the ring beam at the column location and its value relative to the uniform applied load can be termed the stress amplification ratio $\zeta$. The evaluations of the stress amplification ratios $\zeta$ arising from different ring beam stiffness ratios $\psi$ that were identified by Topkaya and Rotter [19] are reproduced in Fig. 3. To ensure that the ring beam is adequately stiff to fulfil its function, design calculations require an upper bound on this relationship, so a convenient empirical upper bound was developed based on the data of Fig. 3, defined by:

$$\zeta = 1 + 0.21\left(3 + \log\left(\frac{\psi}{4}\right)\right)^2 \qquad (1)$$

As shown in Fig. 3 the ring beam stiffness ratio $\psi$ should be quite low to achieve a relatively uniform axial stress above it. This criterion is thus very demanding and usually leads to very big ring beams for typical geometries.

One alternative method of achieving uniform axial membrane stresses is to use an intermediate ring stiffener with a flexible ring beam as shown in Fig. 4 (i.e. double ring arrangement of Fig. 1f). Greiner [20,21], Öry et al. [22] and Öry and Reimerdes [23] showed that an intermediate ring stiffener can be very effective in reducing the circumferential non-uniformity of axial stresses in the shell. Studies conducted by these researchers identified the variation of the axial membrane stress distributions up the height of the shell. It was shown that an intermediate ring stiffener can achieve a dramatic decrease in the peak axial membrane stress, producing a more uniform stress state above the intermediate ring. Recently Topkaya and Rotter [24] showed that there is an *ideal location* for an intermediate ring stiffener, such that the axial membrane stress above this ring is circumferentially completely uniform. The ideal location is identified by the height $H_I$ above the ring beam, defined as the vertical distance between the top of the ring beam and the centre of the intermediate ring stiffener as shown in Fig. 4. This was determined analytically and is expressed in terms of basic geometric variables as follows:

$$H_I = \sqrt{12(1+\nu)}\,\frac{r}{n} \qquad (2)$$

where $n$ = number of uniformly spaced column supports; $r$ = middle surface radius; and $\nu$ = Poisson's ratio. For the case where $\nu = 0.3$, Eq. (2) simplifies further into:

$$H_I = 3.95\frac{r}{n} \approx \frac{4r}{n} \qquad (3)$$

In cases where a shell with large radius rests on a few supports, the