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Free vibration of soft-core sandwich panels with general boundary conditions by harmonic quadrature element method

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ABSTRACT

In this paper, the harmonic quadrature element method is presented for free vibration analysis of soft-core sandwich panels with general boundary conditions. To remove limitations existing in various beam theories, the thin faces are modeled by Euler-Bernoulli beam theory but the soft-core is directly modeled by two-dimensional elasticity theory. A novel harmonic quadrature sandwich panel element with arbitrary number of nodes is developed and the equation of motion is established utilizing Hamilton principle. Explicit formulations are worked out to ease the implementation of the element. To show the applicability of the method, several examples, including two types of core material and five combinations of boundary conditions, are investigated. Results are compared to existing data as well as finite element data of ABAQUS. Comparisons show that the proposed method exhibits a high convergence rate and can yield accurate results in all cases.

1. Introduction

Sandwich structures possess high specific stiffness and strength, as well as excellent energy absorption capability, and thus have been widely applied in aerospace, marine and automobile industries [1]. The static and dynamic behavior of sandwich structures has been caused a lot of attention [2–7].

The static and dynamic behavior of sandwich structures depends largely on the properties of the core, the difference between the properties of the thin face sheets and the core, as well as on the boundary conditions. Many simple and sophisticated theoretical models have been developed thus far. The latest and most sophisticated one is called the extended high-order sandwich panel theory (EHSAPT) [5]. Although most engineering beam theories do not recover all stresses accurately through their constitutive equations [7,8], the EHSAPT can yield all stress components accurately through its constitutive equations under sine-distributed load and simply supported boundary conditions [5]. Perhaps due to the limitation of the pre-assumed through-thickness variation of displacements, however, the EHSAPT [6] and the EHSAPT-based finite element method [9] and quadrature element method (QEM) [10,11] yield less accurate higher mode frequencies. Consider the fact of that the soft-core sandwich panel is essentially a two-dimensional (2D) structure, thus 2D elasticity theory should be employed to get accurate higher mode frequencies. Currently, closed-form 2D elasticity solutions are available only for a simply supported sandwich panel [6], which may provide adequate insight into the physics of the problem and help in checking the accuracy and the efficiency of various numerical methods. For other boundary conditions, numerical methods should be used to obtain the solutions.

Comparing to the strong form methods, such as the mesh-less method [12], discrete singular convolution (DSC) [13,14], differential quadrature method (DQM) [15–18], and harmonic differential quadrature method (HDQM) [19–22], the weak form methods, such as the well-known finite element method (FEM), would be better in handling irregular geometry, discontinuous distribution of load and the force boundary conditions, and thus FEM is widely used in practice. To design safe structures, the dynamic effects should be assessed with precision. The free vibration analysis by the conventional finite element method demands great computational cost to get the accuracy for the higher mode frequencies. Therefore, other accurate and efficient weak form numerical methods should be developed.

Similar to the FEM, the weak form quadrature element method (QEM) is also formulated on the principle of minimum potential energy [23]. The QEM has been proved to possess both the accuracy of global methods and the flexibility of the local method. The major difference from the conventional FEM and the spectral element method (SPE) is that the QEM uses the differential quadrature rule in deriving the element equation in an explicit form that is well suited for implementation in a computer code.

Since the inversion of Vandermode matrix is required in the early version of the QEM, the number of node in one direction cannot be greater than 15 and the number of nodes should be fixed in the

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derivation. To remove these deficiencies, Gauss-Lobatto-Legendre (GLL) nodes and GLL quadrature are used in the formulations of the QEM [10,24–26]. Since the nodes are also the integration points and thus the strain at integration points can be calculated by using the explicit formulas in DQM.

The formulations of the QEM with GLL nodes and GLL quadrature seem exactly the same as the ones of the spectral finite element (SFE) [27,28], and thus the performance of the QEM should be similar to the SFE [23].

Different from the existing EHSAPT-based QEM [11], present sandwich panel element is established based on the combination of Euler-Bernoulli beam theory and 2D elasticity theory. More precisely, the top and bottom thin face sheets are modeled by Euler-Bernoulli beam theory while the core is directly modeled by 2D elasticity theory. In this way, the limitation existing in EHSAPT-based QEM [11] can be removed to ensure the accuracy of higher mode frequencies. Besides, the harmonic differential quadrature rule is used thus the proposed method is called the harmonic quadrature element method (HQEM) to distinguish the existing QEM.

The objective of this article is to establish a novel harmonic quadrature element method for the free vibration analysis of soft-core sandwich panels under general boundary conditions. General formulations for the element with arbitrary number of nodes are provided. In the "Expressions of strain and kinetic energy" section, formulas of the sandwich panel model are given. In the "Formulations of the harmonic quadrature sandwich panel element" section, the explicit expressions of the stiffness and mass matrices are provided. In the "Numerical results and discussion" section, convergence and numerical comparison are made to validate the correctness and accuracy of the present method. Numerical results are also presented and compared to data of ABAQUS for sandwich panels with five combinations of boundary conditions. The paper ends with the "Conclusion" section.

2. Expressions of strain and kinetic energy

The sandwich panel, schematically shown in Fig. 1, has a length of a. The thicknesses of top and bottom faces are f_t and f_b, and the thickness of the core is $2c$. Unit width is assumed to simplify the presentation. The Cartesian coordinate system is also shown in Fig. 1.

Let $\xi = 2x/L - 1$, $\eta = z/c$ and $\xi, \eta \in [-1, 1]$. The strain energy for the core of the sandwich panel element with length L can be given by

$$U = \frac{1}{2}\int_0^L \int_{-c}^{c} \{\varepsilon(x, z, t)\}^T [D] \{\varepsilon(x, z, t)\} dz dx$$
$$= \frac{Lc}{4}\int_{-1}^{-1}\int_{-1}^{-1} \{\varepsilon(\xi, \eta, t)\}^T [D] \{\varepsilon(\xi, \eta, t)\} d\eta d\xi \tag{1}$$

where $\{\varepsilon(x, z, t)\}$ or $\{\varepsilon(\xi, \eta, t)\}$ is the strain vector, and t is the time.

The strain vector are given by

$$\{\varepsilon(x, z, t)\} = \begin{Bmatrix} \varepsilon_x(x, z, t) \\ \varepsilon_z(x, z, t) \\ \gamma_{xz}(x, z, t) \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u(x, z, t) \\ w(x, z, t) \end{Bmatrix} \tag{2}$$

where $u(x, z, t)$ and $w(x, z, t)$ are the displacement components in x and z directions.

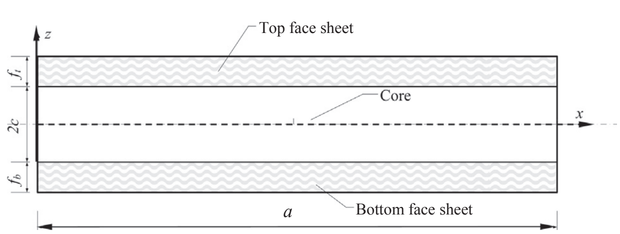

Fig. 1. Sketch of a sandwich panel.

Assume that the core material is homogeneous and orthotropic. The matrix $[D]$ in plane stress condition is given by

$$[D] = \begin{bmatrix} \frac{E_x}{1-\nu_{xz}\nu_{zx}} & \frac{\nu_{xz}E_z}{1-\nu_{xz}\nu_{zx}} & 0 \\ \frac{\nu_{zx}E_x}{1-\nu_{xz}\nu_{zx}} & \frac{E_z}{1-\nu_{xz}\nu_{zx}} & 0 \\ 0 & 0 & G_{zx} \end{bmatrix} \tag{3}$$

where material principal directions coincide with the direction of x and z, E, G, ν are elasticity modulus, shear modulus, and Poisson's ratio, respectively.

To reduce the number of unknown functions, the top and bottom face sheets of the sandwich panel are modeled by Euler-Bernoulli beam theory in terms of the displacements at the interfaces. After using the displacement compatibility conditions at $z = c$ and $-c$, the strain energy for the top and bottom face sheets of the sandwich panel element with length L is given by

$$U^t = \frac{L}{4}\int_{-1}^{1} \left[\frac{du^c(x, t)}{dx}, -\frac{d^2w^c(x, t)}{dx^2} \right] \begin{bmatrix} \hat{A}^t & \hat{B}^t \\ \hat{B}^t & \hat{D}^t \end{bmatrix} \begin{Bmatrix} \frac{du^c(x,t)}{dx} \\ -\frac{d^2w^c(x,t)}{dx^2} \end{Bmatrix} d\xi \tag{4}$$

where $u^c(x, t) = u(x, c, t)$, $w^c(x, t) = w(x, c, t)$, superscript t denotes the top face sheet, $\hat{A}^t$, $\hat{B}^t$, $\hat{D}^t$ are calculated by

$$(\hat{A}^t, \hat{B}^t, \hat{D}^t) = \int_0^{f_t} E^t(1, \hat{z}, \hat{z}^2) d\hat{z} \tag{5}$$

and E^t is the elasticity modulus of the top face sheet.

By the same token, the strain energy for the bottom face sheet of the sandwich panel element with length L is given by

$$U^b = \frac{L}{4}\int_{-1}^{1} \left[\frac{du^{-c}(x, t)}{dx}, -\frac{d^2w^{-c}(x, t)}{dx^2} \right] \begin{bmatrix} \tilde{A}^b & \tilde{B}^b \\ \tilde{B}^b & \tilde{D}^b \end{bmatrix} \begin{Bmatrix} \frac{du^{-c}(x,t)}{dx} \\ -\frac{d^2w^{-c}(x,t)}{dx^2} \end{Bmatrix} d\xi \tag{6}$$

where $u^{-c}(x, t) = u(x, -c, t)$, $w^{-c}(x, t) = w(x, -c, t)$, superscript b denotes the bottom face sheet, $\tilde{A}^b$, $\tilde{B}^b$, $\tilde{D}^b$ are calculated by

$$\left(\tilde{A}^b, \tilde{B}^b, \tilde{D}^b\right) = \int_{-f_b}^{0} E^b(1, \tilde{z}, \tilde{z}^2) d\tilde{z} \tag{7}$$

and E^b is the elasticity modulus of the bottom face sheet.

The kinetic energy for the core of the sandwich panel element with length L and height $2c$ is given by

$$T = \frac{Lc\rho}{4}\int_{-1}^{1}\int_{-1}^{1} [\dot{u}(\xi, \eta, t)^2 + \dot{w}(\xi, \eta, t)^2] d\xi d\eta \tag{8}$$

where ρ is the mass density of the core and the over dot represents the first order derivative with respect to time t.

The kinetic energy for the top face sheet of the sandwich panel element with length L is given by

$$T^t = \frac{L}{4}\int_{-1}^{1} \{I_0^t[(\dot{u}^c)^2 + (\dot{w}^c)^2] - 2I_1^t \dot{u}^c \dot{w}^c_{,x} + I_2^t(\dot{w}^c_{,x})^2\} d\xi \tag{9}$$

where I_0^t, I_1^t, I_2^t are calculated by

$$(I_0^t, I_1^t, I_2^t) = \int_0^{f_t} \rho^t(1, \hat{z}, \hat{z}^2) d\hat{z} \tag{10}$$

and ρ^t is the mass density of the top face sheet.

Similarly, the strain energy for the bottom face sheet of the sandwich panel element with length L is given by

$$T^b = \frac{L}{4}\int_{-1}^{1} \{I_0^b[(\dot{u}^{-c})^2 + (\dot{w}^{-c})^2] - 2I_1^b \dot{u}^{-c} \dot{w}^{-c}_{,x} + I_2^b(\dot{w}^{-c}_{,x})^2\} d\xi \tag{11}$$

where I_0^b, I_1^b, I_2^b are calculated by

$$(I_0^b, I_1^b, I_2^b) = \int_{-f_b}^{0} \rho^b(1, \tilde{z}, \tilde{z}^2) d\tilde{z} \tag{12}$$